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Plunger Inserter Device

Erik Firganek
Olivet Nazarene University, erikfrgnk20@gmail.com

Ben Martin
Olivet Nazarene University, benmartin1522@gmail.com

Christian Mikrut
Olivet Nazarene University, christianmikrut.1@gmail.com

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Plunger Inserter Device

Team: Ben Martin, Christian Mikrut, Erik Firganek
Advisor: Dr. Camilo Giraldo

Thursday, April 21st

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Acknowledgements

- Joshua Martin, sponsor contact and controls engineer for EcoLab
  - Provided necessary information
  - Gave tour of EcoLab and Worksite 520
- Dr. Anne Figus, Department chair for Family and Consumer Science at Olivet
  - Aided team with AutoCAD
- Dr. Camilo Giraldo, Walker School of Engineering Professor
  - Guided the development of Simulink and MATLAB code
  - Researched and completed double-pendulum analysis

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Sponsor Background

- EcoLab - Joliet, IL
- Sponsor Contact: Joshua Martin
- About EcoLab
  - "Global leader in water, hygiene and infection prevention solutions and services that protect people and vital resources."
  - Recognized for long-standing commitment to sustainability
- Purpose
  - A growing market requires higher production
  - EcoLab and Employees benefit

ECOLAB®

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Problem Description

- Ecolab wishes to automate the insertion of plungers into their less-than-gallon “LTG” bottles at Worksite 520
  - Unsatisfying movements for employees
- Initial project scope
  - Part I: Orientate the plunger from manufacturer setting
  - Part II: Insert plunger into LTG bottle
- Change of scope
  - Assume Part I is completed by a future project
  - Assume plungers’ geometry does not variate

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Design Objectives

- System must withstand current and future production
  - System currently filling 8 bottles, want to upgrade to 10 (25% increase)
- Fully automated system
- Working under the assumption that plungers are pre-aligned
- Plunger inserter device must have the ability to run for the duration of all 3 working shifts
- Decrease number of spills, leaks, & packaging defects
- Minimal error rate (< 1%)

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Functional Requirements

- Plungers must be partially pressed into each LTG bottle
- 8 to 10 bottles must be completed between 32 and 45 seconds
- Future ladder program must be compatible with existing software that EcoLab uses

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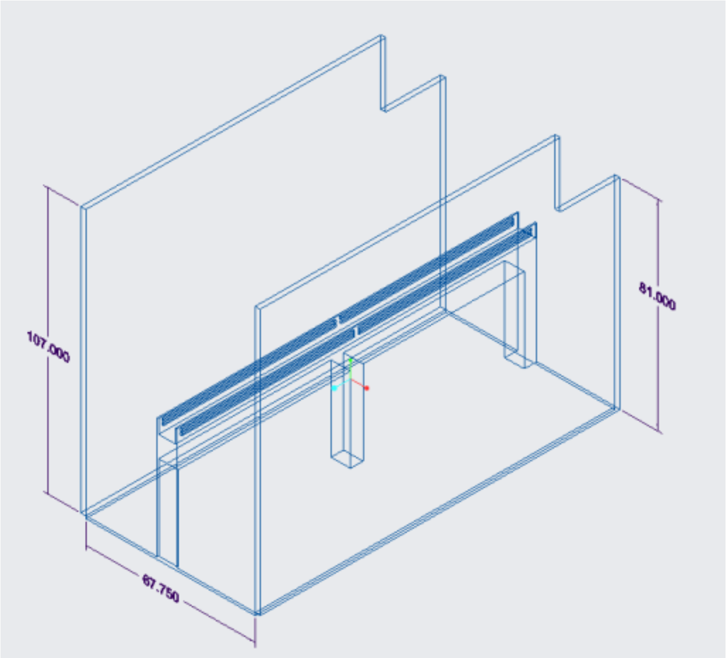
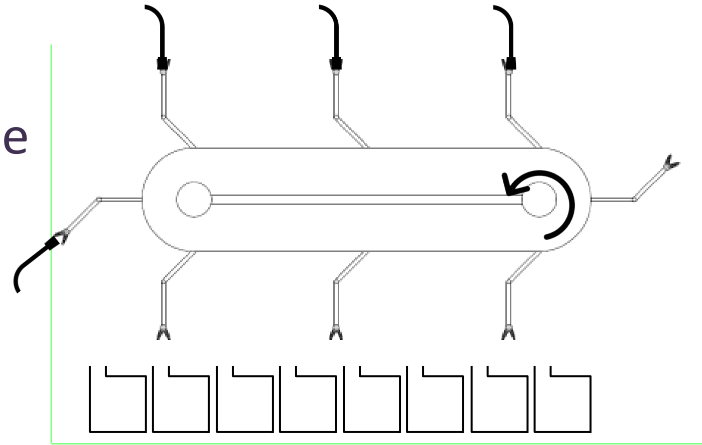
Design Constraints

- Space constraint
- Chemicals in Worksite 520 vary in pH levels (1 to 13); thus, material constraints are present
  - 316 stainless steel, IP65 electrical cables, and other polymers resistant to range of pH levels
- Final product must be powered by less than 480 Volts and 90 PSI of compressed air
- Total cost must be less than $500k



Design Alternatives - The Oval

- Fixed above assembly line, drive train rotates arms and inserts plungers
- Grippers grab properly orientated plunger from above
- At the bottom, a 2-link system would insert the plunger into the LTG bottles
- Comments
  - Multiple moving pieces, likely requires more maintenance
  - Expensive
  - Requires the plunger to be initially up-side-down



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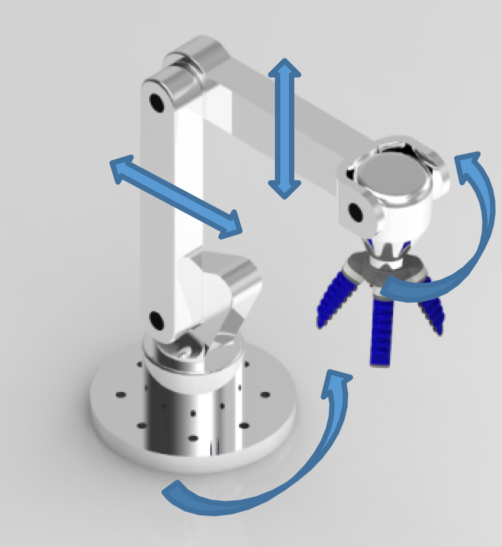
Design Alternatives - Robotic Arm

- Robotic arm would be positioned next to assembly line
  - 4 degrees of freedom, plus the grabber
- Robot grabs a properly orientated plunger, and inserts it into an LTG bottle
- Comments:
  - Compact design
  - Complex design and analysis (i.e., 3D dynamics)
  - Multiple robots would be needed



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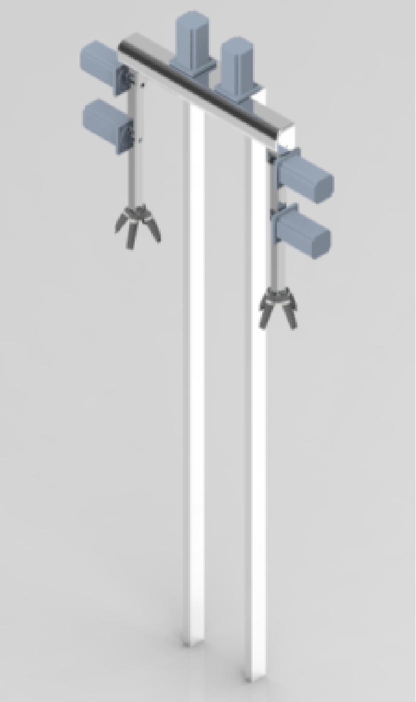
Proposed Design - Gateway Track Arms

- Two gateways mounted over the conveyor belt
  - Two robotic arms with 3 degrees of freedom, plus the gripper
  - Positioned next to plungers
- Robotic arm grabs a properly orientated plunger, and inserts it into an LTG bottle
- Comments:
  - Compact design
  - Two gateways would be used



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Design Selection

- Decision Matrix based on Safety, Space, Timing, Complexity, and Cost
- Through evaluation, the Gateway Track Arms had the highest total

Decision Matrix

	Safety (5)	Complexity (2)	Space (4)	Timing (3)	Cost (1)	Total Score
Robotic Arm	5	3	3	2	3	52
Gateway Track Arms	4	4	3	5	3	58
The Oval	3	1	1	3	1	31

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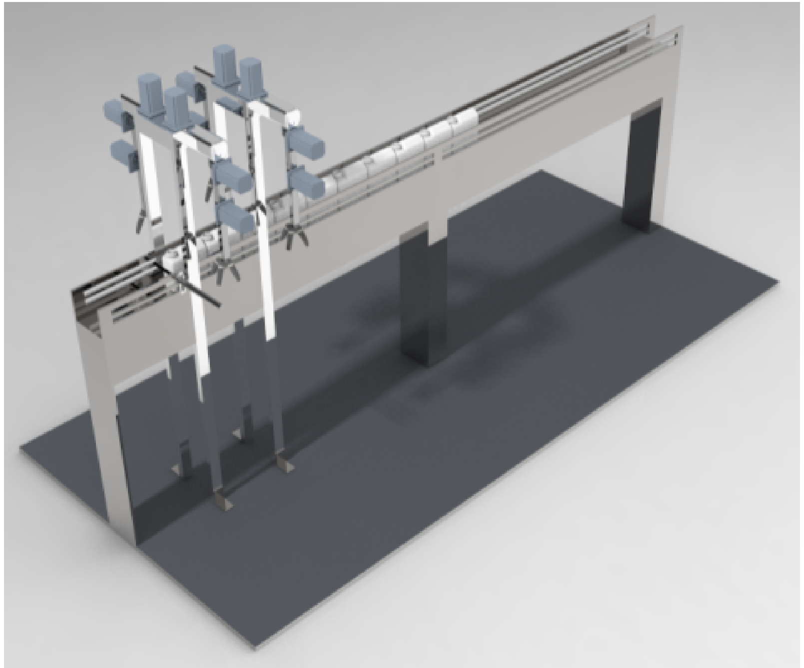
Gateway Track Arms

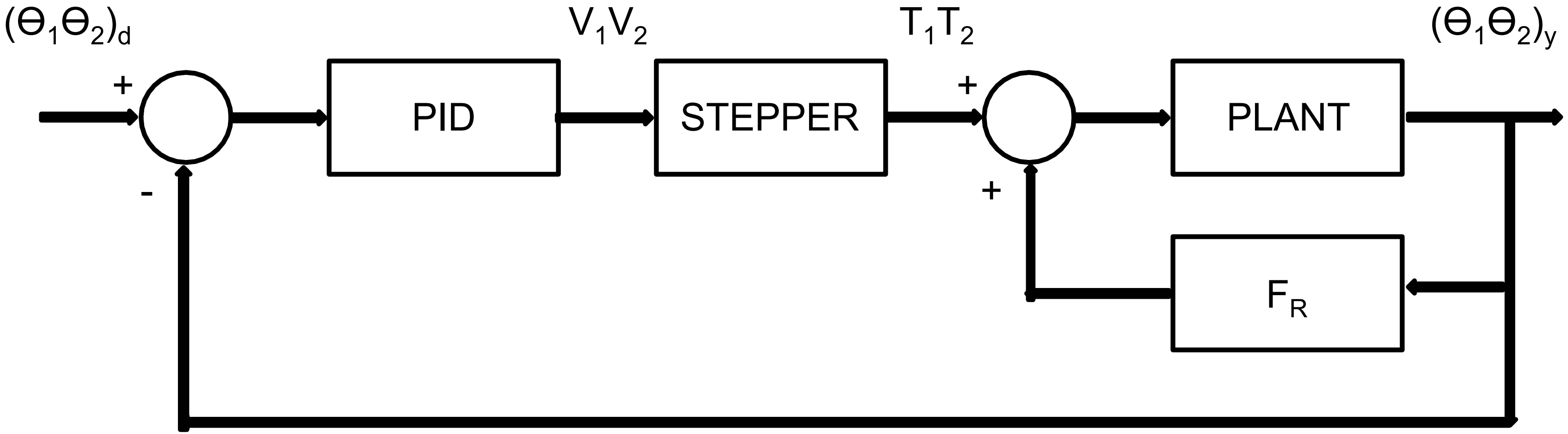
- Gateways mounted to the ground
  - Line is susceptible to vibrations
- LTG Bottles are placed at the same location every time
  - Pneumatic stopping arm located in front the Gateways



Design Validation

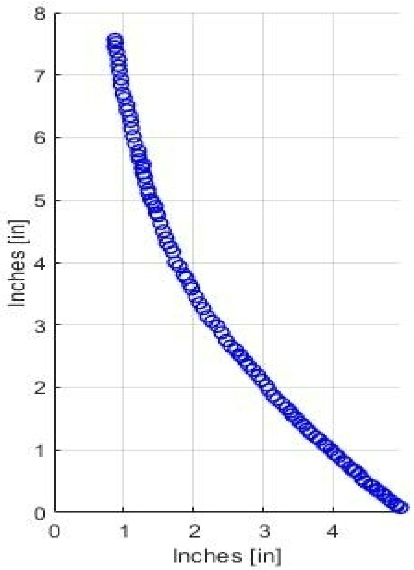
- Validated
  - Materials through inspection
  - Stepper motors exert enough torque to operate arms
  - 11N force exerted
  - Partial insertion of plungers
  - 10 LTG bottles per cycle
  - Fully automated
- Not Validated
  - Operate for 3 shifts (Durability)
  - Error rate < 1%

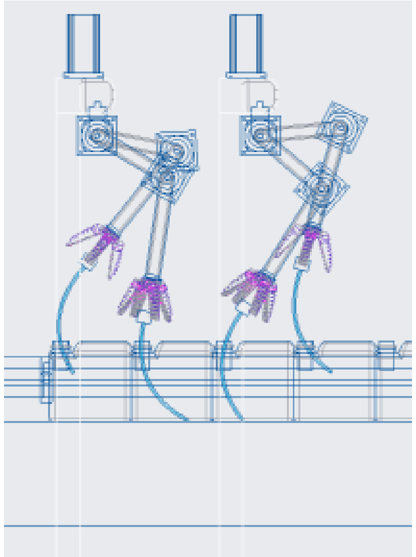
Generalized Feedback Loop



Gateway Track Arms Analysis

- Obtained plunger geometry
  - Used MATLAB "grabit.m" code to analyze the curvature of the plunger
- Plunger geometry → Gateway arm motion
  - Motion for inserting plunger requires plunger geometry
  - Limits of motion can be simplified with angles
  - Four main phases:
    - Load phase – arm grabs plunger
    - Transport phase – arm hovers directly over LTG bottle
    - Insertion phase – arm inserts plunger
    - Reset phase – arm returns to origin





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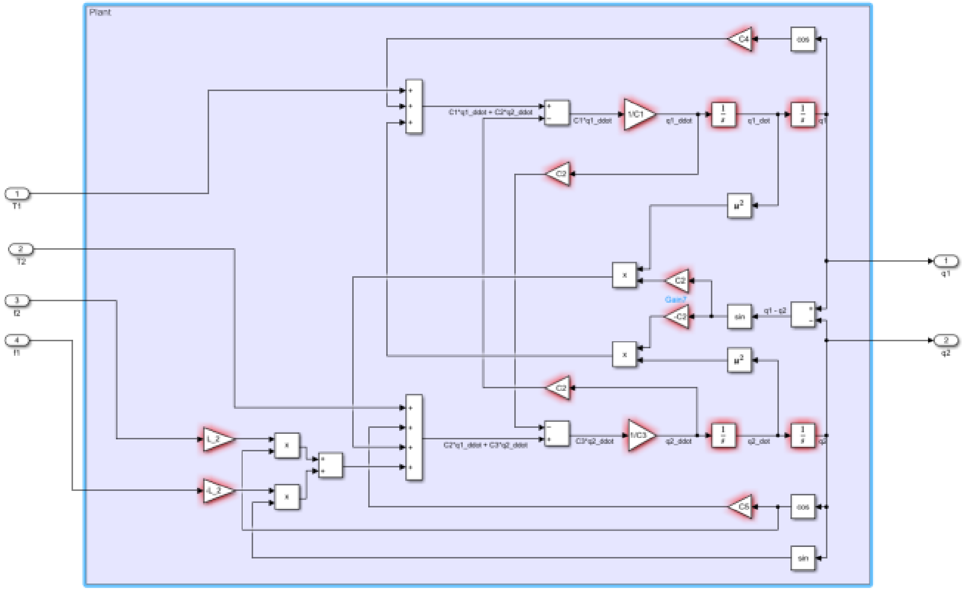
Plant Function

- Modeled robotic arm as a double compound pendulum
- Derived a differential equation to model the pendulum
- Constraints: Gravity, mass, length, mass moment of inertia and mass center
- Transfer function inputs torque and outputs angles



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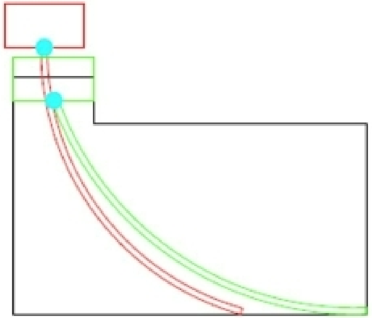
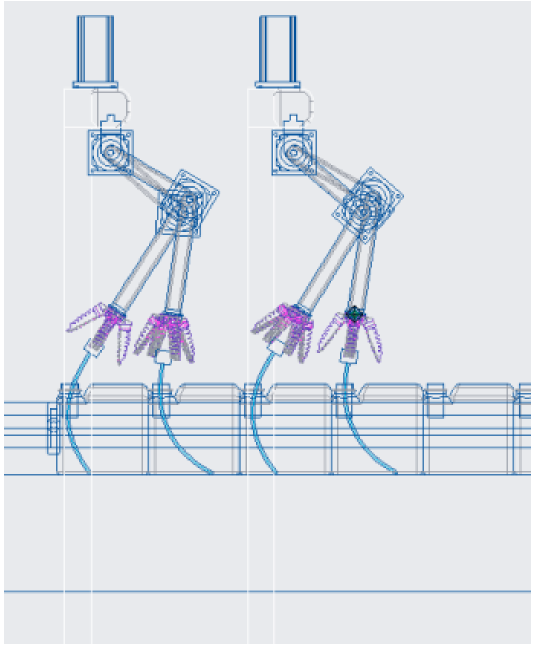
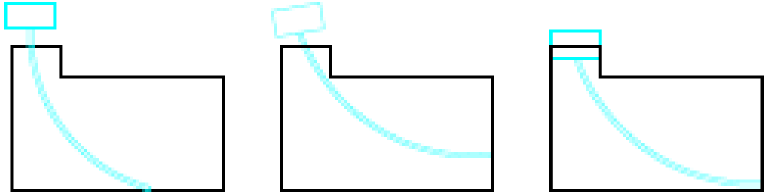
Insertion Phase

- Touchdown zone
  - 3 touchdowns
  - Assume constant 45° angle once force is initiated
  - Assume constant force and constant resistance







Force Analysis Test

- Force output required to partially insert plunger into LTG bottle
- 11N required

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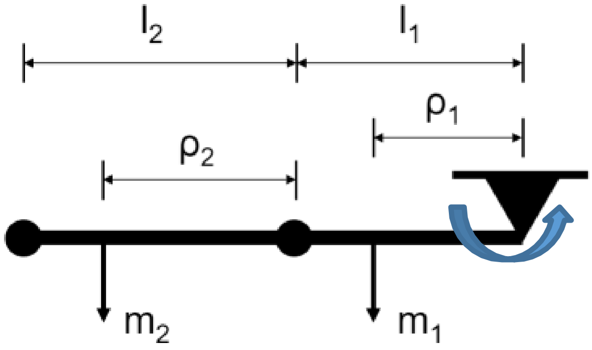
Maximum Torque Analysis

- Motor sizing and torque output
  - High torque to weight ratio
  - Max torque 10.49 Nm
- Optimization of torque
  - Hollow arms
  - Two motors per arm



m_1 [kg]	4.3499
m_2 [kg]	4.3953
ρ_1 [m]	0.0126
ρ_2 [m]	0.0147
l_1 [m]	0.2159
l_2 [m]	0.2286

$$\Sigma M = m_1 g \rho_1 + m_2 g (l_1 + \rho_2) - M_s$$
$$M_s = m_1 g \rho_1 + m_2 g (l_1 + \rho_2)$$
$$M_s = T_{max}$$
$$\boldsymbol{T_{max} = m_1 g \rho_1 + m_2 g (l_1 + \rho_2)}$$

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Motor Analysis

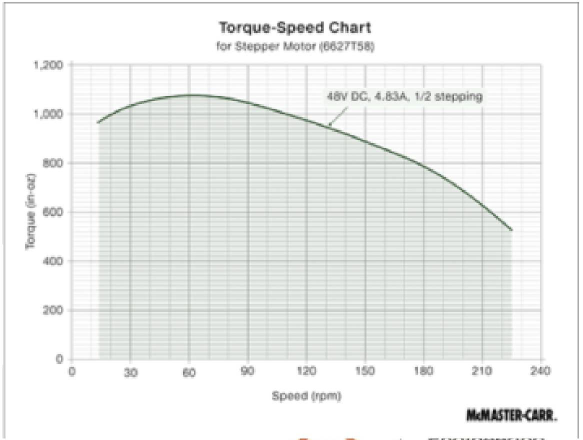
- Motor modeled with DC motor equation
- Motor values taken from McMaster-Carr
- Damping value assumed from *Control Systems Engineering 8th Edition*
- Maximum torque 10.13 Nm

$$T_{\text{stall}} = \frac{K_t}{R_a} e_a$$

$$\boxed{K_b = \frac{e_a}{\omega_{\text{no-load}}}}$$

$$\boxed{T_m = -\frac{K_b K_t}{R_a}\omega_m + \frac{K_t}{R_a} e_a}$$

Component	Motor
Motor Frame Size	NEMA 34
Maximum Holding Torque	1,435 in.-oz.
Maximum Speed	600 rpm
Maximum Current per Phase	4.83 A
Resistance per Phase	0.85 ohms
Inductance per Phase	9.61 mH
Full Step Increment	1.8°
Rotor Inertia	21.9 oz-in.^2



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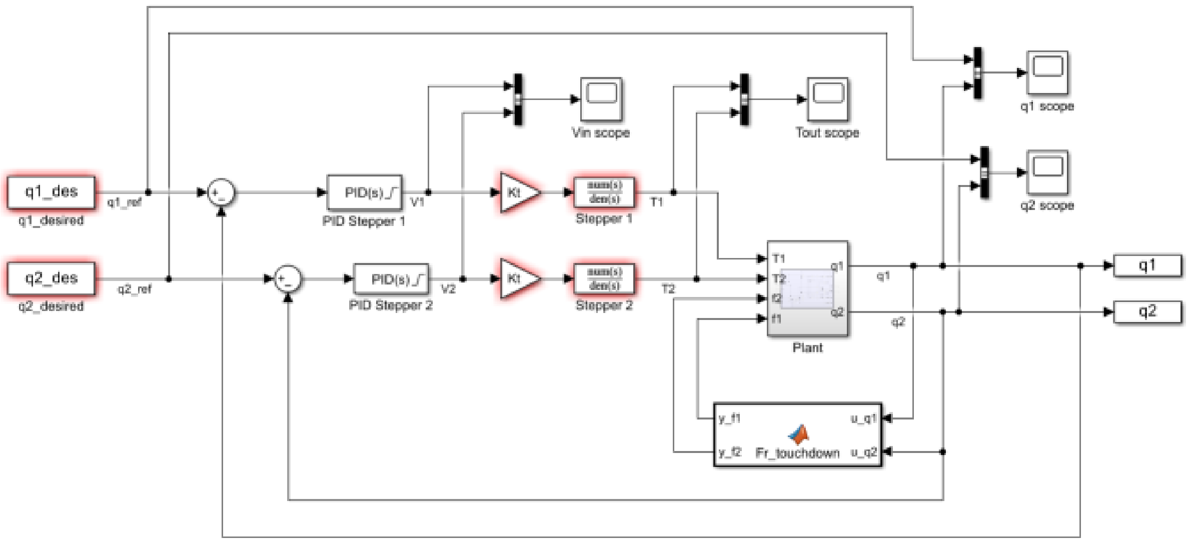
Final Simulink Control System



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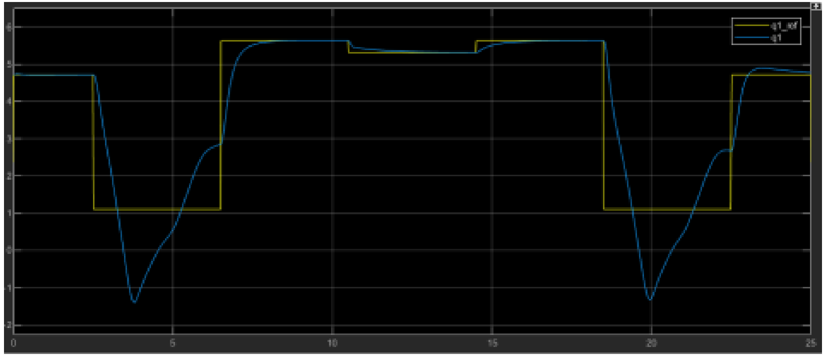
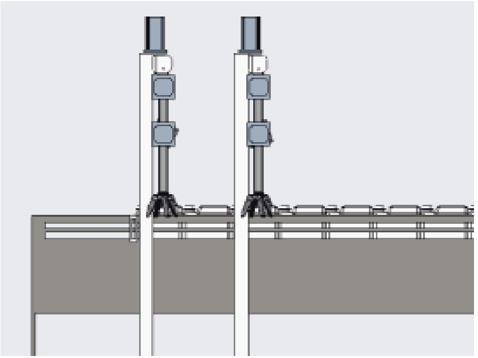
Modeling Arm Motion

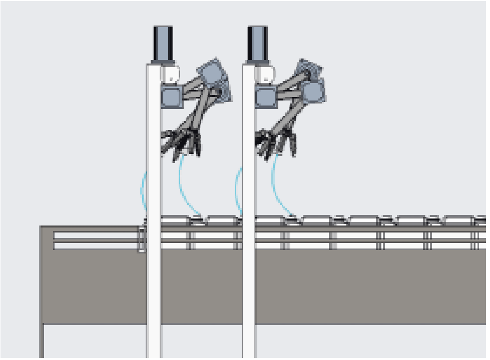


Link 1 Simulation of Position Vs Time

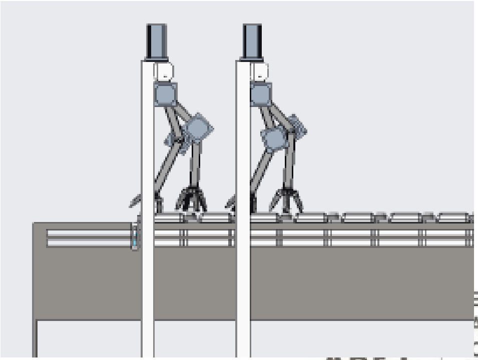
Position 1: -90°



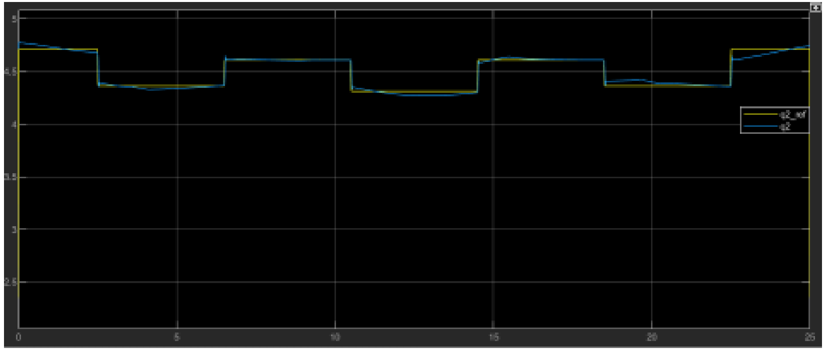
Position 2: 63°



Position 3: -56°

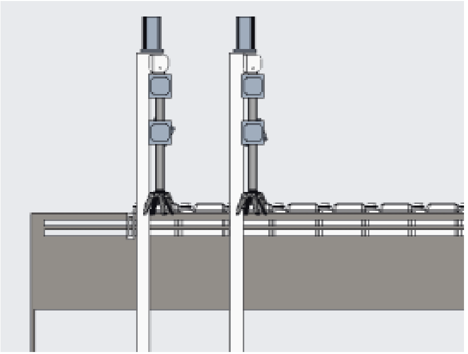


Modeling Arm Motion Cont.

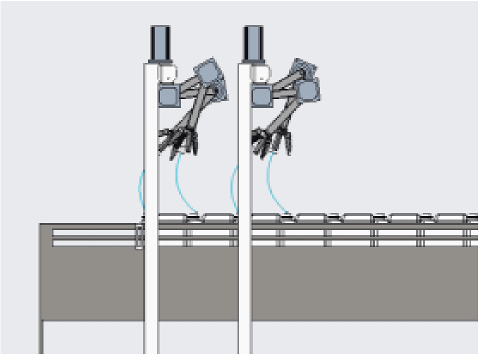


Link 2 Simulation of Position Vs Time

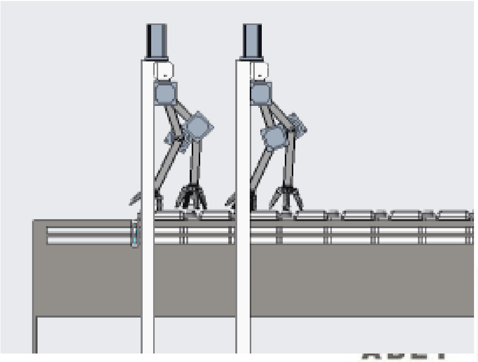
Position 1: -90°



Position 2: -110°



Position 3: -113°



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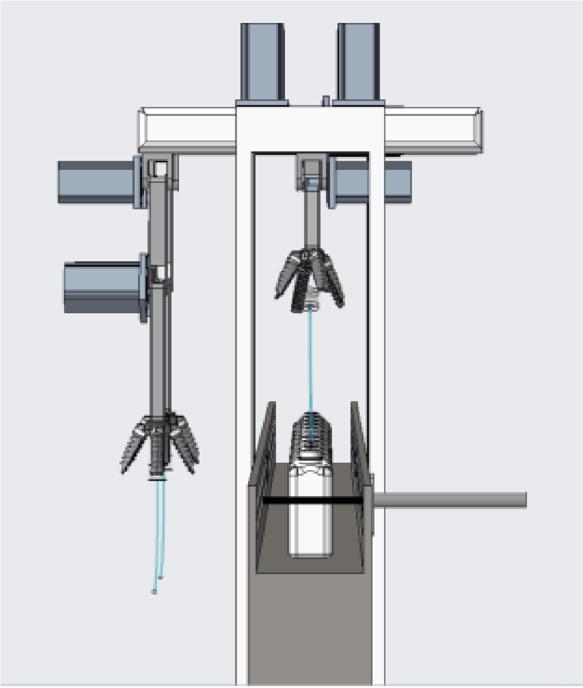
Design Validation

- 10 LTG bottles per cycle
  - Validated through Simulink
- Fully automated
  - Validated through Simulink



Conclusion

- Benefits
  - Reallocate employees
  - Increased production by 25%

- Recommendations & Future Steps
  - Create Creo Parametric simulations
    - Longevity/Durability and error rate analysis

References

- Norman S. Nise, Control Systems Engineering 8th Edition (2019)
- Gary T. Yamaguchi, Dynamic Modeling of Musculoskeletal Motion (2006)
- Gordon, Michael E. Planar N-Link Open Chain. PhD. Dissertation. Stanford University (1990)
- Unchained Robotics
- McMaster-Carr

Q&A

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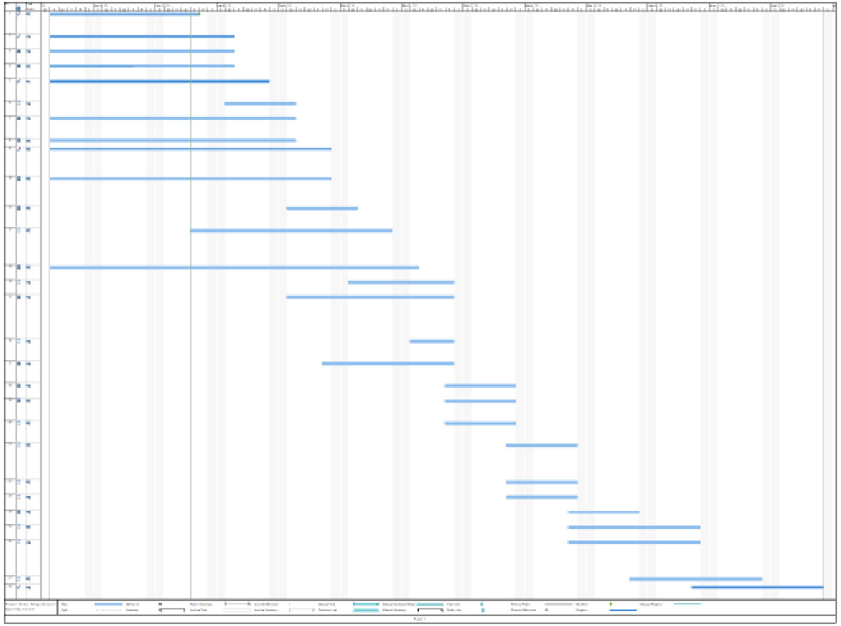
Schedule

		Task Mode	Task Name	Duration	Start	Finish
1	✓		Obtain Coordinates for Curvature of Plunger	13 days	Tue 1/11/22	Thu 1/27/22
2	✓		XY Plot of Curvature of Plunger	15 days	Tue 1/11/22	Mon 1/31/22
3			Double Compound Pendulum	15 days	Tue 1/11/22	Mon 1/31/22
4			List of Providers on Steppers	15 days	Tue 1/11/22	Mon 1/31/22
5	✓		CAD of Conveyor Belt, 8 to 10 LTG Bottles, and Stopper	19 days	Tue 1/11/22	Fri 2/4/22
6			Double Compound Pendulum + Stepper	6 days	Mon 1/31/22	Mon 2/7/22
7			Research on How Steppers are Modeled	20 days	Tue 1/11/22	Mon 2/7/22
8			Design Experiment	20 days	Tue 1/11/22	Mon 2/7/22
9	✓		Add Gateways (grounded) in the Final Location to CAD	24 days	Tue 1/11/22	Fri 2/11/22
10			Report to Teams the Length of Arms and Their Location with Respect to Stopper	24 days	Tue 1/11/22	Fri 2/11/22
11			Double Compound Pendulum + Stepper + Resistive Forces	6 days	Mon 2/7/22	Mon 2/14/22
12			Through "claymation" in Creo, Determine Position of Arms so Plunger is Inserted	17 days	Thu 1/27/22	Fri 2/18/22
13			Conduct Experiment and Analyze Data	30 days	Tue 1/11/22	Mon 2/21/22
14			Final Difference Equation	10 days	Mon 2/14/22	Fri 2/25/22
15			Mass, torque, response time, accuracy, sensitivity, cost, modeling equations and power requirements	15 days	Mon 2/7/22	Fri 2/25/22
16			Report Results in technical writing format	5 days	Mon 2/21/22	Fri 2/25/22
17			Translate motion findings in terms on theta_1 and theta_2	11 days	Fri 2/11/22	Fri 2/25/22
18			Summarize all CAD work	6 days	Fri 2/25/22	Fri 3/4/22
19			Consult with Josh for conveyor belt inputs and outputs	6 days	Fri 2/25/22	Fri 3/4/22
20			Simulink code for (theta1,theta2) = f(T1,T2,Fr)	6 days	Fri 2/25/22	Fri 3/4/22
21			Gather and Combine as much information in CAD that communicates final design	6 days	Fri 3/4/22	Fri 3/11/22
22			Design logic of entire operation	5 days	Mon 3/7/22	Fri 3/11/22
23			Add steppers to simulink code	6 days	Fri 3/4/22	Fri 3/11/22
24			Add PID controllers to simulink code	6 days	Fri 3/11/22	Fri 3/18/22
25			List limitations and next steps	11 days	Fri 3/11/22	Fri 3/25/22
26			Consult with Josh, Prof. Erikson, and others on how a ladder code is written	11 days	Fri 3/11/22	Fri 3/25/22
27			Tune PID controllers	11 days	Fri 3/18/22	Fri 4/1/22
28	✓		Write and test ladder code	11 days	Fri 3/25/22	Fri 4/8/22



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Extra Slide for Force Analysis

Trial for Maximum Force (Ben Martin)	Mass [Kg]
1	1.2
2	1.3
3	1.3
4	0.9
5	1.2

Trial for Maximum Force (Christian Mikrut)	Mass [Kg]
1	1.2
2	1
3	1.1
4	0.9
5	1.1

Combined Average (Christian & Ben)	1.12

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MATLAB Extra Slide

Editor - C:\Users\bamartin\Downloads\EcoLab_Code_Final.m

EcoLab_Code_Final.m

```matlab
%EcoLab Team - ENGR401 and ENGR402
%Olivet Nazarene University
%Last Update: March 16, 2022

clear; close all; clc;

%% General Information on Rigid Bodies
%Gravity [m/s^2]
g=9.81;

%%% Link 1
%Masse [kg]
m_link_1=1.15*0.453592;
m_step_1=8.44*0.453592;
m_1=m_link_1+m_step_1;

%Lenght [m]
L_1=8.5*0.0254;

%Mass center [m]
ro_link_1=0.5*L_1;
ro_step_1=0;
ro_1=(m_link_1*ro_link_1+m_step_1*ro_step_1)/m_1;

%Mass moments of inertia [m^2*kg]
I_1=(1/12)*m_link_1*L_1^2+m_link_1*(0.5*L_1-ro_1)^2+m_step_1*ro_1^2;
    %If mass moment of inertia is found from Creo, edit here

%%% Link 2
%Masse [kg]
m_link_2=1.25*0.453592;
m_step_2=8.44*0.453592;
m_2=m_link_2+m_step_2;

%Lenght [m]
L_2=9*0.0254;

```

```matlab
%Mass center [m]
ro_link_2=0.5*L_2;
ro_step_2=0;
ro_2=(m_link_2*ro_link_2+m_step_2*ro_step_2)/m_2;

%Mass moments of inertia [m^2*kg]
I_2=(1/12)*m_link_2*L_2^2+m_link_2*(0.5*L_2-ro_2)^2+m_step_2*ro_2^2;
    %If mass moment of inertia is found from Creo, edit here

%% General Information on Stepper Motors
%Resistnace [ohms]
R=0.85;

%Inductance [Henry]
L=9.41*10^-3;

%Rotor mass moment of inertia [kg*m^2]
J=21.9;                       %[oz*in^2]
J=J*0.0283495*0.0254^2;       %[kg*m^2]

%Damping rotation [N-m-s/rad]
D=2;

%Rated voltage [V]
Ea=48;

%Holding torque or T_stall [N-m]
T_stall=1435;                           %[oz*in]
T_stall=T_stall*0.278014*0.0254;      %[N-m]

%Maximum angular velocity [rad/s]
omega_NL=600;                 %[rpm]
omega_NL=omega_NL*2*pi/60;   %[rad/s]

%Motor constants
Kt=T_stall*R/Ea;          %[N-m/A]
```

```matlab
Kb=Ea/omega_NL;           %[V*s/rad]

%% Desired Angles
%Time vector [s]
t_0=0; delta_t=0.01; t_f=45;
t=(t_0:delta_t:t_f)';

%Angle steps for arm 1 [rad]
q1_1=deg2rad(63);
q1_2=deg2rad(360-37.7);
q1_3=deg2rad(360-56);
q1_end=3/2*pi;

%Angle steps for arm 2 [rad]
q2_1=deg2rad(360-110);
q2_2=deg2rad(360-96);
q2_3=deg2rad(360-113);
q2_end=3/2*pi;

%Creating q1_des [rad]
q1_des=q1_end*heaviside(t)-q1_end*heaviside(t-4)+...
    q1_1*heaviside(t-4)-q1_1*heaviside(t-8)+...
    q1_2*heaviside(t-8)-q1_2*heaviside(t-12)+...
    q1_3*heaviside(t-12)-q1_3*heaviside(t-16)+...
    q1_2*heaviside(t-16)-q1_2*heaviside(t-20)+...
    q1_1*heaviside(t-20)-q1_1*heaviside(t-24)+...
    q1_2*heaviside(t-24)-q1_2*heaviside(t-28)+...
    q1_3*heaviside(t-28)-q1_3*heaviside(t-32)+...
    q1_2*heaviside(t-32)-q1_2*heaviside(t-36)+...
    q1_1*heaviside(t-36)-q1_1*heaviside(t-40)+...
    q1_end*heaviside(t-40)-q1_end*heaviside(t-45);
q1_des=[t,q1_des];

%Creating q2_des [rad]
q2_des=q2_end*heaviside(t)-q2_end*heaviside(t-4)+...
    q2_1*heaviside(t-4)-q2_1*heaviside(t-8)+...
```

MATLAB Extra Slide

```
        q2_2*heaviside(t-8)-q2_2*heaviside(t-12)+...
        q2_3*heaviside(t-12)-q2_3*heaviside(t-16)+...
        q2_2*heaviside(t-16)-q2_2*heaviside(t-20)+...
        q2_1*heaviside(t-20)-q2_1*heaviside(t-24)+...
        q2_2*heaviside(t-24)-q2_2*heaviside(t-28)+...
        q2_3*heaviside(t-28)-q2_3*heaviside(t-32)+...
        q2_2*heaviside(t-32)-q2_2*heaviside(t-36)+...
        q2_1*heaviside(t-36)-q2_1*heaviside(t-40)+...
        q2_end*heaviside(t-40)-q2_end*heaviside(t-45);
q2_des=[t,q2_des];

%% Simulink Model
%Modeling Constants
C1=m_2*L_1^2+m_1*ro_1^2+I_1;
C2=m_2*L_1*ro_2;
C3=m_2*ro_2^2+I_2;
C4=-g*(m_2*L_1+m_1*ro_1);
C5=-g*m_2*ro_2;

%Unitial conditions
q1_dot_0=0;      %[rad/s]
q1_0=3/2*pi;      %[rad]
q2_dot_0=0;      %[rad/s]
q2_0=3/2*pi;      %[rad]

%Calling model
sim('EcoLab_Sim.slx');

%Turning angles in positions
x_1=L_1*cos(q1);
y_1=L_1*sin(q1);
x_2=x_1+L_2*cos(q2);
y_2=y_1+L_2*sin(q2);
```

```

%Plotting desired vs. actual angles
figure('units','normalized','outerposition',[0 0 1 1]);

subplot(1,2,1); plot(t,q1_des,tout,q1); xlim([0 t_f]);
xlabel('t [s]'); ylabel('q_1 [rad]'); grid;
title('Link 1 Rotation'); legend('Desired','Actual');

subplot(1,2,2); plot(t,q2_des,tout,q2); xlim([0 t_f]);
xlabel('t [s]'); ylabel('q_2 [rad]'); grid;
title('Link 2 Rotation'); legend('Desired','Actual');

% %Plotting simulation
% for ii = 1:length(tout)
%
%     %Setting up figure
%     if ii == 1
%         figure('units','normalized','outerposition',[0 0 1 1])
%     end
%
%     %Spatial simulation
% %     if mod(t(ii),0.01) == 0
%         subplot(2,2,[1 3]);
%         plot([0 x_1(ii)],[0 y_1(ii)],'r'); hold on;
%         plot([x_1(ii) x_2(ii)],[y_1(ii) y_2(ii)],'b'); hold on;
%         axis([-1.25*(L_1+L_2) 1.25*(L_1+L_2) -1.25*(L_1+L_2) 1*(L_1+L_2)]);
%         grid; axis square; title(['t = ' num2str(tout(ii)) ' s']);
%         xlabel('x [m]'); ylabel('y [m]'); pause(0.01);
%         if ii ~= length(tout)
%             clf;
%         end
% %     end
%
%     %Rotation time series
%     if ii == length(tout)
%
%     end
%
% end

```

Motor Analysis Extra Slide

$$\left[\frac{R_a}{K_t}(J_m s + D_m) + K_b\right] s\theta_m(s) = E_a(s)$$

$$T_{\text{stall}} = \frac{K_t}{R_a} e_a$$

$$\boxed{T_m = -\frac{K_b K_t}{R_a}\omega_m + \frac{K_t}{R_a} e_a}$$

$$\boxed{K_b = \frac{e_a}{\omega_{\text{no-load}}}}$$

$$\boxed{\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}}$$

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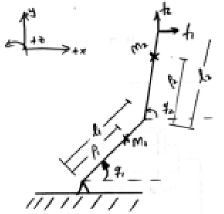
Double Pendulum Analysis



↳ $m_1 \rightarrow$ Arm 1 + Stepper 1, $m_2 \rightarrow$ Arm 2 + Stepper 2 $\Big\}\; n = 2$

↳ Auxiliary Variables $j = 1$; $k = 1, 2$

- $\mu_1 = l_1 \sum_{i=2}^{2} m_i \Rightarrow \mu_1 = l_1 m_2$
- $\mu_2 = 0$

- $\delta_1 = \mu_1 + \dfrac{I_1 + m_1 \rho_1^2}{l_1} = m_2 l_1 + \dfrac{I_1 + m_1 \rho_1^2}{l_1}$

  $\delta_2 = \mu_2 + \dfrac{I_2 + m_2 \rho_2^2}{l_2} = \dfrac{I_2 + m_2 \rho_2^2}{l_2}$

- $\gamma_1 = \mu_1 + m_1 \rho_1 = m_2 l_1 + m_1 \rho_1$

  $\gamma_2 = \mu_2 + m_2 \rho_2 = m_2 \rho_2$

↳ Mass Matrix:

$$\begin{bmatrix} l_1 \left(m_2 l_1 + \dfrac{I_1 + m_1 \rho_1^2}{l_1}\right) & l_1 (m_2 \rho_2) \cos(q_1 - q_2) \\ l_1 (m_2 \rho_2) \cos(q_1 - q_2) & l_2 \dfrac{I_2 + m_2 \rho_2^2}{l_2} \end{bmatrix}$$

↳ N matrix

$$\begin{bmatrix} 0 & l_1 (m_2 \rho_2) \sin(q_1 - q_2) \\ -l_1 (m_2 \rho_2) \sin(q_1 - q_2) & 0 \end{bmatrix}$$

↳ Torque $\begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$

↳ G array $\begin{bmatrix} -g (m_2 l_1 + m_1 \rho_1) \cos(q_1) \\ -g (m_2 \rho_2) \cos(q_2) \end{bmatrix}$

↳ $V_k = -N \begin{bmatrix} \dot{q}_1^2 \\ \dot{q}_2^2 \end{bmatrix}$

$$V = -\begin{bmatrix} 0 & l_1 (m_2 \rho_2) \sin(q_1 - q_2) \\ -l_1 (m_2 \rho_2) \sin(q_1 - q_2) & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1^2 \\ \dot{q}_2^2 \end{bmatrix}$$

$$V = \begin{bmatrix} -l_1 (m_2 \rho_2) \sin(q_1 - q_2)\, \dot{q}_2^2 \\ l_1 (m_2 \rho_2) \sin(q_1 - q_2)\, \dot{q}_1^2 \end{bmatrix}$$

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Double Pendulum Analysis

↳ External force

$$E = \begin{bmatrix} 0 \\ l_2 \left(f_2 \cos(q_2) - f_1 \sin(q_2) \right) \end{bmatrix}$$

$\boxed{\text{All}}$

$$\begin{bmatrix} \overset{C_1}{\cancel{m_2 l_1^2 + m_1 \rho_1^2 + I_1}} & \overset{C_2}{\cancel{m_2 l_1 \rho_2}} \cos(q_1 - q_2) \\ \overset{C_2}{\cancel{m_2 l_1 \rho_2}} \cos(q_1 - q_2) & \overset{C_3}{\cancel{I_2 + m_2 \rho_2^2}} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} + \dots$$

$$\begin{bmatrix} \overset{C_4}{\cancel{-g(m_2 l_1 + m_1 \rho_1)}} \cos(q_1) \\ \overset{C_5}{\cancel{-g(m_2 \rho_2)}} \cos(q_2) \end{bmatrix} + \begin{bmatrix} \overset{C_2}{\cancel{-m_2 l_1 \rho_2}} \sin(q_1 - q_2) \dot{q}_2^2 \\ \overset{C_2}{\cancel{m_2 l_1 \rho_2}} \sin(q_1 - q_2) \dot{q}_1^2 \end{bmatrix} + \dots$$

$$\begin{bmatrix} 0 \\ l_2 f_2 \cos(q_2) - l_2 f_1 \sin(q_2) \end{bmatrix}$$

↳ Is are with respect to mass center

↳ Constants:

- $C_1 = m_2 l_1^2 + m_1 \rho_1^2 + I_1$
- $C_2 = m_2 l_1 \rho_2$
- $C_3 = m_2 \rho_2^2 + I_2$
- $C_4 = -g(m_2 l_1 + m_1 \rho_1)$
- $C_5 = -g m_2 \rho_2$

↳ Equations:

$$\begin{bmatrix} C_1 & C_2 \\ C_2 & C_3 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} + \begin{bmatrix} C_4 \cos(q_1) \\ C_5 \cos(q_2) \end{bmatrix} + \begin{bmatrix} -C_2 \sin(q_1 - q_2) \dot{q}_2^2 \\ C_2 \sin(q_1 - q_2) \dot{q}_1^2 \end{bmatrix} + \dots$$

$$+ \begin{bmatrix} 0 \\ l_2 f_2 \cos(q_2) - l_2 f_1 \sin(q_2) \end{bmatrix}$$

↳ Masses and moments of inertia

↳ $m_1 = m_{link\,1} + m_{step\,1}$

↳ $\rho = \dfrac{\Sigma m \rho}{\Sigma m} = \dfrac{m_{link} \frac{1}{2} l + m_{step}(0)}{m_{link} + m_{step}}$

$$\rho = \frac{m_{link}\, l}{2(m_{link} + m_{step})}$$

↳ $I = \frac{1}{3} m_{link} l^2 + m_{link} \left(\frac{1}{2} l - \rho \right)^2 + m_{step} \rho^2$

①

②

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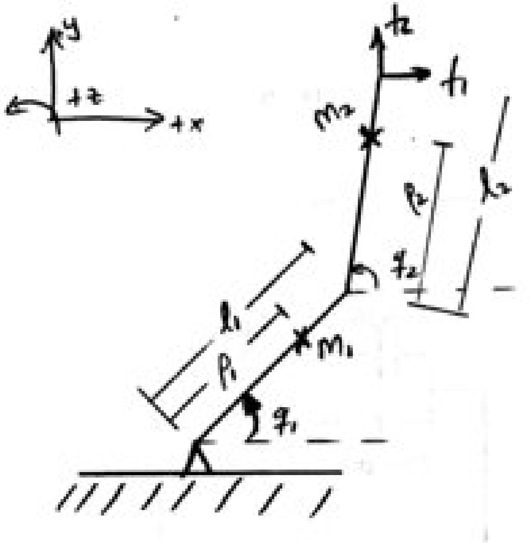
Double Pendulum Analysis Model



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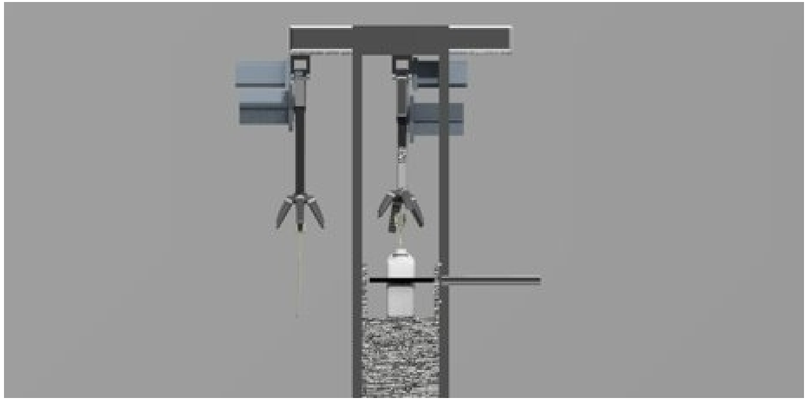

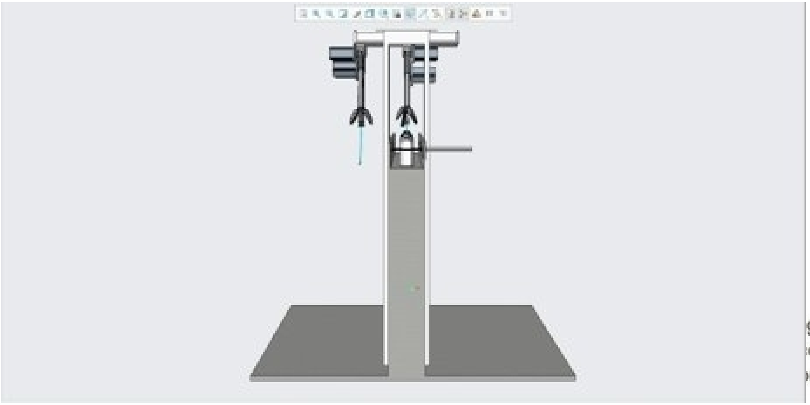

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