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Implementing Common Core State Standards for Mathematics Through Lesson Study

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Implementing Common Core State Standards for Mathematics Through Lesson Study

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ABSTRACT

The Common Core State Standards for Mathematics (CCSSM) represent the beginning of a new era in American education. For the first time, a majority of states are sharing expectations for student knowledge in mathematics. While standards cannot change education, the means by which these standards are implemented contribute to the mathematical achievement of students. For instance, the CCSSM incorporate separate content and practice standards for students. Content standards are familiar to most educators, but the expectation of developing mathematical skills highlighted in the practice standards will require changes to lesson preparation and teaching.

In an effort to provide pre-service and in-service teachers a model for implementing the CCSSM in the classroom, the following project used a descriptive lesson study involving content and practice standard pairings. The lesson study was conducted in a geometry classroom in an Illinois high school during the spring of 2013. After examining the development of standards in modern education, a rationale and methodology is developed to implement standard pairings. The lessons and reflections on teaching are included to provide evidence of the possibilities and challenges of implementing the Common Core State Standards for Mathematics. Finally, a discussion of the lesson study is provided with suggestions for further research.

Keywords: Common Core State Standards for Mathematics, CCSSM, Mathematics Education, Standards, Lesson Study, Descriptive Research, Content Standards, Practice Standards, High School, Secondary Mathematics, Geometry, Geometric Constructions, Pre-service Teachers, Lesson Plans, Discussion.

INTRODUCTION

The philosophies, strategies, and methods of teachers continually change as they develop experience and knowledge of teaching. On a larger scale, schools and districts continually evaluate curricula as needed with the goal of improving student achievement. Heightening the complexity of the American education system, state governments and the U.S. Department of Education introduce and abandon initiatives throughout time. All of these factors affect education in the United States individually, but these factors also interact to impact classroom instruction. A recent development in the American education system that exemplifies this impact on classroom instruction is the adoption of the Common Core State Standards for Mathematics (CCSSM). These standards are the result of a consortium of governors, educational researchers, and policymakers reacting to the evidence of mediocre mathematics achievement in the United States. While the standards themselves cannot change education, the CCSSM are already impacting instruction through their mandated use in 45 states and their connection to upcoming standardized tests. Teachers in states undergoing implementation are required to align instruction to these standards; however, implementing the CCSSM is not a simple matter of changing citations in lesson plans. With the inclusion of the Standards for Mathematical Practice, the CCSSM requires teachers to consider the development of mathematical skills in their students. These practice standards require careful consideration when planning instruction, as well as attention throughout the teaching of a lesson and subsequent reflection. The following project addresses the development of the

CCSSM and provides an example of implementing these standards through the use of a lesson study model.

REVIEW OF LITERATURE

The Evolution of Standards in American Education

Standard is a commonly used word in education, but its meaning has changed with time. When defining this term in relation to the classroom, the National Research Council (2002) noted “To many educators, a ‘standard’ is a statement describing what a person should know or be able to do” (p. 17). Basically, a standard is a knowledge or skill that students should develop at a given point his education career. From this definition, a standard is any objective a teacher uses in the classroom to guide his teaching and from whichstudent learning is measured. These standards may be simple or complex, requiring various instruction techniques and supports/scaffolding for students. In this sense, standards are a natural part of the classroom; however, this basic view of standards has changed with the development of state and national standards. This development occurred over the past half-century as evidence emerged indicating a lack of mathematical knowledge among U.S. students. For example, in a review of the Third International Math and Science Study (TIMSS) exam, researchers Schmidt, Houang, and Cogan (2002) discovered“By the end of secondary school our [U.S. student] performance is near the bottom of the international distribution. In both math and science, our typical graduating class outperformed students in only two other countries… This is serious” (p. 12). From the results of this wide reaching study of mathematical knowledge around the world, the authors found that U.S. students consistently lagged behind their international peers. This performance relates in part to classroom instruction, resulting in attention to standards in education. As educational author Robert Rothman (2012a) noted, "Standards-based reform has been the de facto national education reform strategy for more than two decades. Spurred by federal legislation, states have placed standards…at the center of their improvement efforts" (p. 12). Reacting to data displaying poor knowledge of mathematical understanding in U.S. students and reports of weak job preparedness in new workers, politicians and educators targeted standards as a cause and solution to problems in student knowledge. By requiring all teachers to implement uniform standards in a state, the author's conclusion is legislators and educational stakeholders hope to improve student knowledge of mathematics by having consistently high expectations. While teachers lose some independence under mandated standards, the goal is to improve teaching and learning in general. Consequently, standards have been transformed from a simple part of the classroom into a broad plan for educational excellence in the United States. Overall, the use of standards has evolved over the past decades in the United States.

Although state legislators recognized the importance of standards in the last quarter of the twentieth century, the shift from state standards to the Common Core State Standards was a recent change. Sincethe release of test results and the landmark writing *A Nation at Risk*, author Lynn Arthur Steen (2007) noted, "nearly all states have established content standards in mathematics" (p. 88). After recognizing the declining mathematical knowledge of students and the potential of standards, states implemented unique sets of standards to guide instruction and state assessments. While this action had merit, the existence of 50 unique sets of standards for mathematics created different problems. After instituting the National Assessment of Educational Progress (NAEP) in all states as part of No Child Left Behind, politicians and educators became aware of inconsistencies between state standards. Researcher Robert Rothman (2012b) observed the results of the NAEP revealed "that in some states nearly all students reached proficiency on state tests, while only a handful reached that level on the NAEP… These findings suggested that some states were setting standards too low" (p. 59). From the data compiled through the NAEP, educators realized that students in different states learned different content at varying levels of proficiency. Along with this finding, a bleak picture of education was illustrated through the lowering of proficiency benchmarks and standards. Rather than improving education throughout the United States, state standards were resulting in positive and negative changes in student mathematics achievement. In response to these shocking results, many people inside and outside of the education community called for national standards. The work of researchers David Coleman and Jason Zimba (2007) embodied this call to action by advocating fewer standards that are written in easily understood language and raise expectations of student achievement. While the researchers avoided promoting national standards in their work, this summary obviously points towards higher standards for all students in the United States. With NAEP results and the work of researchers in mind, governors began working with educators to develop national standards. The outcome of this collaboration was the release of the Common Core State Standards for Mathematics (CCSSM) in 2010 and subsequent adoption by 45 states. In summary, the development of the CCSSM was the result of standards becoming a means to improve education and an act to remedy to negative effects of state standards.

CCSSM: Emphasizing Content and Practice

The Common Core State Standards for Mathematicsare grounded in the moderate success of previous standard documents. The strongest similarity between the CCSSM and previous standard efforts is the function of the documents. The Common Core State Standards Initiative (2010) emphasized that the CCSSM "do not dictate curriculum or teaching methods" (p. 5). Just as existing state standard documents noted, the authors of the CCSSM acknowledge that methods of instruction and organization are ultimately the decision of the teacher. While standards may direct some of the desired objectives of lessons, teachers must decide how to guide students to these objectives through a variety of instructional practices. From this understanding, the CCSSM allows creative teaching to exist as it did during prior standard movements. Another example of a similarity in the CCSSM and existing standards is found in the use of practice standards. Along with content standards that specify mathematical concepts and procedures for students to master, the CCSSM includes practice standards. In defining practice standards, the Common Core State Standards Initiative (2010) stated, "The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on… the NCTM [National Council of Teachers of Mathematics] standards" (p. 6). From the organization's statement, the CCSSM incorporate ideas for practice standards from an existing document crafted by the NCTM. This shared emphasis on skills and mathematical thinking situates the CCSSM in the continuing efforts to improve mathematical learning in the United States. The similarity between the CCSSM and NCTM standards is furthered evidenced in recent guides developed to assist teachers implementing the standards. Educators Susan O'Connell and John SanGiovanni (2013) prefaced their book about incorporating the practice standards into instruction by summarizing "The CCSS[M] and NCTM Standards both value content and process. The CCSS[M] emphasize that the content standards should be blended with their Standards for Mathematical Practice, which are closely related to NCTM's Math Process Standards" (p. 7). From this observation, the Common Core standards embrace the successful aspects of previous standard documents. When comparing the balance of content and practice in the documents, the CCSSM may be seen as revision or summary of the NCTM standards (although these standards were never state mandated). Critics of the CCSSM often question the importance of the briefly stated practice standards when critiquing the document, but the inclusion of such standards can be supported by and elaborated upon using the NCTM standards. When considering some of the key statements from the authors of the CCSSM, the new standards exist within the continuum of mathematics standards in the United States.

In contrast to these similarities, the CCSSM differs from existing state standards in a variety of ways. For example, the Common Core State Standards Initiative (2010) firmly addressed differences in the CCSSM by summarizing "These Standards are not intended to be new names for old ways of doing business.

They are a call to take the next step. It is time for states to work together to build on lessons learned from the two decades of standards based reform" (p. 5). From this use of figurative language, the CCSSM are established as a departure from existing standards and a development based upon the failures of these documents. The CCSSM is not meant to nationalize existing standards, but to improve and focus standards for classroom instruction. Based upon the guiding principle of the document, the CCSSM is a new development in education standards. Another difference between the CCSSM and current standards that is of particular importance to educators is the general degree of focus in the CCSSM. Evidence of this difference is found in the conclusion of researchers Andrew Porter, Jennifer McMaken, Jun Hwang, and Rui Yang (2011), who noted "The Common Core standards represent considerable change from what states currently call for in their standards and in what they assess. The Common Core standards are somewhat more focused in mathematics" (p. 114). Upon reviewing many standard documents, the observation of the authors is that the CCSSM are a general improvement over prior efforts in relation to focus. This finding is significant, as the degree of focus within the CCSSM relates to the ability to interpret and apply standards in the classroom. Since the standards are more focused, there exists the possibility of greater success in the implementation and attainment of the standards. In all, the Common Core State Standards for Mathematics exhibit differences from prior standard documents in the United States.

Illinois and the CCSSM

In relation to Illinois, the CCSSM are an improvement upon the Illinois Learning Standards (ILS) for mathematics. A great example of this improvement is found in the previously mentioned studyabout focus in the documents. When grouping the standards in the CCSSM and respective state documents into content cells, researchers Porter, McMaken, Hwang, and Yang (2011) found that the CCSSM required 94 groups and the ILS required 157. From this finding, the CCSSM is clearly more focused than the Illinois Learning Standards. Instead of a teacher dealing with a scattering of content categories under the ILS, the CCSSM offers a greater number of standards that can be grouped in fewer categories and taught in larger cohesive units. From this evidence, the CCSSM are an improvement in focus that teachers in Illinois will appreciate when crafting lessons and curricula. In addition to this numerical study, additional evidence illustrates the improvement of the CCSSM over the Illinois Learning Standards. When analyzing the documents with a simple grading scheme, the educational researchers at the Thomas B. Fordham Institute (2010) found "Illinois's mathematics standards are among the worst in the country…the CCSS math standards are vastly superior" (p. 118). From the organization's strong statement, the adoption of the CCSSM by Illinois will raise the expectations of student learning for its students. These improved standards in turn will promote excellence in instruction and support of learning, resulting in the overall improvement of mathematical knowledge in Illinois's students. Overall, the CCSSM are an improvement upon the Illinois Learning Standards for mathematics.

PROJECT RATIONALE

While the implementation of the CCSSM is occurring in schools throughout the United States, resources and information for teachers must be developed to insure student success. For instance, the results of an Illinois State Board of Education (n.d.) survey about the Common Core that revealed only 13.5% of teachers in the state feel completely prepared to implement the standards. Hypothetically, teachers in Illinois and other states are already working on implementation of the standards. The results indicate that teachers require resources and examples of implementing the CCSSM in the classroom. If teachers feel ill equipped to implement both content and practice standards, they will resort to teaching to the test over teaching to build the necessary skills indicated in the CCSSM. Former high school teacher Darcy Ireland captured the need for a study when she noted that her school only gave her a flipchart containing the CCCSM and few examples of using the standards in the classroom (personal communication, April 24, 2012). In order to achieve the goals of the Common Core, educators need access to examples of instruction that incorporate both types of standards. When speaking about this aspect of implementation, NCTM President Linda Gojak (2013a) concludes:

> If we are to realize the potential of the Common Core, teachers and administrators must have access to high-quality professional development, including opportunities to deeply understand the Standards for Mathematical Content and the implications for instruction of the Standards for Mathematical Practice.” (n.p)

From this statement, lessons that incorporate both sets of standards will be valuable for any teacher seeking to implement the mathematics standards effectively. Developing lessons that model those standards serve to promote intentional and effective implementation of the CCSSM. With this understanding, it is obvious that example lessons that embody the standards are necessary to effective implementation.

METHODOLOGY

Pairing Content and Practice Standards

Based upon the CCSSM document and other sources, a foundation for developing lessons rested in pairing content standards with appropriate practice standards. Educator Jan Christianson (2012) emphasizes this pairing when concluding, "To successfully implement the Common Core content standards, it will be necessary for teachers to fully implement the practice standards" (p. 74). The observation of the author sets a precedent for implementing the CCSSM by calling for equal emphasis on practice and content standards. High quality lessons that emulate the Common Core necessarily incorporate both parts of the document. EducatorSusan Russell (2012) upheld this implementation structure in a similar fashion by summarizing, "The Standards for Mathematical Practice focus on what it means to do mathematics… they are necessarily embedded in content" (p. 52). Because of the general nature of the practice standards, teachers must be active in incorporating these standards and recognize opportunities to attain them when fulfilling traditional content standards. The practice standards advocate an active role for teachers and students in evaluating how they are thinking about mathematics; therefore, high quality lessons necessarily involve the practice standards. From an understanding of the importance of pairing content and practice standards, high quality lessons were developed for the study.

Overview of Lesson Study Model

With a foundation for developing lessons, a model was required for evaluating the success of implementing the chosen standards. While databasedmeasures will undoubtedly be used to evaluate the impact of the CCSSM on student learning, implementation requires a descriptive form of evaluation to help teachers discover the success and challenges of teaching with the standards. For this reason, a lesson study model was used when teaching lessons that paired content and practice standards. Mathematics education researchers Elizabeth Burroughs and Jennifer Luebeck (2010) summarized the essence of the model when observing “Lesson study is a process for creating deep and grounded reflection about the complex activities of teaching that can be shared and discussed with other members of the profession” (p. 391). From this summary, lesson study is a powerful form of descriptive research that allows educators to develop resources for other teachers and to engage in professional development. In relation to the CCSSM, a lesson study model provided a means of rigorously reviewing the possibilities and pitfalls of pairing standards during instruction. Along with this advantage, the components of lesson study provided a systematic approach to implementing paired content and practice standards. While lesson study often takes many forms, researchers Lynn Hart, Deborah Najee-ullah, and Karen Schultz (2004) condensed the essential aspects of lesson study as planning, teaching, and reflecting. During the planning stage, the goals for a lesson are established and the related mathematics is explored. The teaching of a lesson occurs in a normal classroom setting with one or more teachers observing the progress of the lesson. Finally, reflection upon the lesson occurs through written and oral formats. Math teacher Penelope Tolle (2010) highlighted the importance of reflection on lessons by noting "The postlesson discussion provides the group [of educators] with an open forum to evaluate that work with a focus on student learning" (p. 183). From this summary, the reflection stage of lesson study provides a way to qualitatively measure the success of the lesson. The emphasis on student learning ultimately relates back to lesson objectives and chosen standards, making this form of research helpful for implementing the CCSSM. With the means to qualitatively measure the success of a lesson, using the lesson study model when implementing the CCSSM allows teachers to determine the extent that chosen content and practice standards appear in students. From this knowledge, teachers are equipped to improve instruction and effectively implement standards. Along with these practical motives for choosing a lesson study model for this project, the use of descriptive lesson studies are beneficial in developing the teaching abilities of in-service and pre-service teachers. In the article previously referenced, Burroughs and Luebeck (2010) found that pre-service teachers who participated in lesson study developed significant knowledge "about teaching, learning, and collaboration in a mathematics community" (p. 398). Basically, careful implementation of a lesson study involving a pre-service teacher allows him to develop various competencies required for a mathematics classroom. In connection to the goals of this study, using a lesson study modeled maximized the opportunities for the author to develop teaching expertise and practical knowledge of the CCSSM. Based upon the foundation of pairing practice and content standards, the lesson study model allows the success of implementation to be qualitatively measured.

IMPLEMENTATION

Background About School, Classroom, and Lesson Study Arrangement

After establishing a foundation for lessons and a method for inquiry, a lesson study involving standard pairings was conducted. The study was conducted in a geometry classroom at a high school in Illinois. The placement of the pre-service teacher in this school was in conjunction with a field experience required by a university course for the spring of 2013. According to the latest Illinois State Board of Education (2013) Report Card for the district, the high school serves 2067 students in grades 9-12 through a staff of 110 teachers. The developed lessons were taught during two sections of the geometry course, including a section cotaught with a special education teacher. The classes consisted of students in grades 10-12. Various exceptionalities were exhibited among the students (requiring the support of a special educator teacher) and at least one student was classified as an English Language Learner. Three lessons were developed by the pre-service teacher under the supervision of a cooperating teacher and including consultation with the university project mentor. Lessons were taught under the observation of the cooperating teacher, along with additional oversight provided by university supervisors. Throughout the study, reflection discussions were completed via conversations between the author, cooperating teacher, and supervisor. In addition, the pre-service teacher completed written reflections using a consistent battery of questions (see Table 1). With preliminary background information revealed, one is prepared to review the implementation of the defined methodology.

Table 1: Teacher Reflection Questions. Adapted from Lynn Hart, Deborah Najee-ullah, and Karen Schultz (2004).
1. *How did the lesson go overall?*
2. *What worked well? What didn't work well?*
3. *How well do you feel the desired Standard for Mathematical Practice and paired Content Standard were implemented?*
4. *Did students seem engaged in activities related to the chosen standards and objectives?*
5. *What could be done differently to improve the effectiveness of this lesson?*

Plan: Topic of Instruction and Rationale

During the planning phase of the study, the pre-service and cooperating teachers decided to teach geometric constructions. This choice was directly connected to the content standardG-CO.12; however, the topic was chosen because of its connections to various practice standards. As a form of mathematical proof, geometric constructions are a topic where students are expected to justify their reasoning.In describing this requirement, mathematician James R. Smart (1998) summarizes that "The problem in a construction is not simply that of drawing a figure to satisfy certain conditions but whether… a theoretically exact solution can be obtained" (p. 211). From this summary, the process of completing a construction relies upon justifying that a solution exists using geometric properties such as congruence and related theorems. Thisact is characteristic of Standard for Mathematical Practice 3 (construct viable arguments and critique the reasoning of others). Beyond generating an accurate sketch of a construction, students must be able to verify each step in their process using congruence or a basic axiom of geometry. Another aspect of constructions that aligns with a practice standard is the ability to apply constructions to new situations. This ability embodies Standard for Mathematical Practice 1 (make sense of problems and persevere in solving them). For example, students were guided through the process of copying an angle with the expectation that the construction would be used later to form a parallel line. Finally, the ability to represent constructions through paper foldsresulted in the choice of Standard for Mathematical Practice 5 (use appropriate tools strategically). As will be discussed later, this practice standard was not the best choice for this topic. Based upon the nature of the topic and the ability to pair standards, the pre-service and cooperating teacher chose to use geometric constructions for the focus of the lesson study.

Teach and Reflect: Lessons Created for the Study

Appendix A provides the lesson plans created for the study with the accompanying reflections completed by the pre-service teacher. Each lesson was taught twice during the school day. After teaching one section of the geometry course, a short discussion between the cooperating and pre-service teachers was held to discuss possible alterations to lesson presentation. Using the knowledge from this discussion, the lesson was taught a second time for another section of the course. Each instructional period consisted of 55 minutes of teacher-led activity and group time for students to work with constructions. Following the second presentation of the lesson, a longer discussion occurred between the pre-service and cooperating teacher that included the questions from Table 1. During one of these discussions, a university supervisor was also present to provide feedback about a lesson. Finally, the uniform battery of questions outlined in Table 1 was used as the pre-service teacher completed written reflections for each lesson. The following section provides a discussion the entire lesson study.

DISCUSSION

Throughout the course of the lesson study, the cooperating and pre-service teacher observed variations in student comprehension of content and use of the targeted practice standards. These variations among students are to be expected in any math classroom, so this result is not necessarily an indicator of a successful or unsuccessful lesson. The variation in student use of the targeted practice standard is also notable because such differences highlight the need for patience in implementing the CCSSM. The practice standards focus on critical thinking skills that are developed as a student learns mathematics throughout schooling; therefore, educational professionals need to understand that the skills embedded in these standards will require much use and time to develop. NCTM president Linda Gojak (2013b) highlights this need for patience in a recent statement about the CCSSM, when she concludes "successful mathematics achievement involves a deep understanding of concepts that goes beyond memorizing procedures that do not make sense, and this shift will take time. A few years will not be enough time to measure success" (n.p). From this statement, the reasoning skills outlined in the practice standards will not be as easily attained as paired content standards. Before a teacher decides to abandon the explicit use of practice standards in lessons, he/she must be reminded of the gradual nature of the practice standards. Every student reasons differently, so the appearance of the practice standards in a math classroom will never be uniform. As author Marilyn Burns (2012) emphasizes in her discussion of the practice standards, “It’s important not to think about ‘fixing’ students who don’t demonstrate particular skills of understanding because partial understanding and confusion are part of the learning process-students learn in their own ways, at their own paces” (p. 46). A teacher must be committed to developing the skills advocated in the practice standards, while understanding that these skills will develop differently in each of his/her students. The given statement highlights what was seen throughout the lesson study as students demonstrated varying degrees of success in explaining their reasoning. Some students were more comfortable using reasoning skills, while other students experienced the partial confusion associated with the development of mathematical reasoning. Basically, the lesson study reinforces the well-known fact any successful math teacher understands: Every student develops problem-solving skills uniquely and over time as teachers maximize opportunities to actively reason with mathematics. This fact supports the choice to actively consider the practice standards related to content as a teacher plans and carries out instruction. By designing lessons that target content and practice standards, a teacher is more aware of the reasoning required of students and creates activities to reinforce these skills. Overall, the variations in student demonstration of practice standards and comprehension of content are consistent with the realities of implementing the CCSSM.

Another finding from the lesson study was that the model is helpful for developing experience implementing the CCSSM. When developing the content and practice standard pairings, the pre-service teacher gained valuable knowledge in interpreting the meaning of the practice standards by determining if any of them applied to the chosen content standard. This planning process is consistent with the recommendations of Susan Jo Russell (2012), who advocates that math teachers "must identify content in the curriculum where a teaching-learning emphasis on each practice [standard] can most productively occur" (p. 52). From this recommendation, placing the practice standards in one's lesson planning cannot be a matter of choosing a random standard to pair with content. A teacher must be deliberate in targeting a practice standard for a lesson, so that it is connected to understanding the content of a lesson. Pairing practice standards with content standards may require multiple attempts as the realities of the classroom make a given pairing unhelpful for students. For instance, pairing practice standard 5 with the content standard in the third lesson ended up creating confusion for students when the lesson was taught due to uncertainties that resulted in directions. In revising the lesson for future use, the standard pairing may still be useful if the ordering of activities or use of tools is altered. The lesson study revealed that pairing practice and content standards during planning makes a teacher consider the thinking required to learn mathematics, while adding to his/her understanding of the CCSSM. Along with this benefit, the lesson study encouraged the use of reflective thinking when teaching lessons aligned to the CCSSM. Throughout the lesson study, the pre-service and cooperating teacher discussed the effectiveness of each lesson in relation to the chosen standard pairing. Beyond general questions about the overall quality of a lesson, the discussions and written reflections mandated in the lesson study developed the reflective thinking of the pre-service teacher. The pre-service teacher became aware of the challenges and possibilities that exist when teaching lessons with standards pairing, while he developed reflective skills that are valuable for any future instruction. This result reflects the findings of other lesson studies involving pre-service teachers. In the conclusion of a lesson study by researchers Burroughs and Luebeck (2010), the authors note "pre-service teacher involvement in lesson study has clarified the true capabilities of pre-service teachers as reflective and collaborative pre-professionals" (p. 399). By involving pre-service teachers in lesson study, aspiring educators develop valuable skills that will aid them as first-year teachers. In connection to the focus of this lesson study, teachers engaging in similar studies involving standard pairings will grow to understand the complexities of emphasizing the skills of the practice standards in the classroom. From the lesson study conducted, the model provided an effective means of implementing the CCSSM by encouraging active interpretation of standards and reflective thinking about optimal use in the classroom.

Considering the nature of the lesson study conducted, strengths and weakness of using lesson study for implementing the CCSSM are observable. In terms of strengths, the discussion above already noted that using lesson study to implement CCSSM contributed to teacher knowledge. Along with this aspect of the study, the outline of the model (plan with a standard pairing, teach, then reflect) is simple enough for any teacher to use individually or with a group of educators as part of professional development. In relation to the use of standard pairings, an important weakness exists. When reviewing all of the content standards that exist in the CCSSM, each standard may not have a related practice standard that forms an effective pairing. This finding introduces an important limitation for generalizing the given lesson study model for implementing the Common Core. Basically, standard pairings cannot be used in every lesson; thus, teachers will need to development discernment regarding what content is conducive to emphasizing a practice standard. Another weakness of the study is the absence of quantitative data to assess the effectiveness of using lesson study. This limitation is due in part to the intent of lesson study, which is to provide qualitative data for teachers to improve their instruction. In regards to implementing the CCSSM, lesson study does not provide quantitative data for administrators and teachers to assess the extent to which the model is supporting student learning. Essentially, lesson study must be one part of a teacher's approach or a school's plan for improving student learning using the CCSSM. Overall, the strengths and weakness of using lesson study for implementing the Common Core situate the model as one component of an overall plan of implementation.

SUGGESTIONS FOR FURTHER RESEARCH

Upon examining the outcomes of this study and the nature of the CCSSM, numerous research opportunities exist beyond the scope of this project. For example, additional lesson studies may be conducted involving different content and practice standard pairings. Along with this suggestion, additional resources for creating standard pairings can be developed to assist educators in implementation of the CCSSM. The effectiveness of lesson study may be examined if a researcher conducts a longitudinal study of teachers using the model as they implement to Common Core. In terms of the examining the practice standards, observational studies may be developed to provide authentic examples of how these standards appear in various grade levels as the cognitive development of a student progresses. Additionally, a foil to this project may be conducted to assist teachers implementing the Common Core State Standards for English Language Arts. The features of such a lesson study would involve pairing content standards with related College and Career Readiness Anchor standards. Finally, the effectiveness and validity of the Common Core can be addressed by developing quantitative studies or through analyzing data from aligned standardized tests after a few years of data have accumulated. Any or all of these avenues of research will contribute to literature about lesson study and the CCSSM, while generating evidence for future revision of the CCSSM. The Common Core was never intended to be a static document. On the contrary, the writers of the standards held the philosophy that the best current educational practices should shape the CCSSM. Naturally, the results of qualitative and quantitative studies involving the Common Core will provide insight into best educational practices. Robert Rothman (2011) emphasized this arrangement by summarizing that “Once the standards are implemented, researchers must examine them to determine if they are indeed valid... Once this research is available, it is likely the standards will be revised” (p. 99). From this summary, exploration of any facet of the Common Core by qualitative or quantitative means will provide evidence for or against the validity of a particular part of the document. If teachers and educational researchers conduct additional lesson studies with the CCSSM, the results benefit the teachers involved and provide qualitative evidence of the validity of a particular standard. Essentially, numerous research opportunities exist for implementing the CCSSM using lesson study or examining the impact of the standards upon instruction and student achievement.

CONCLUSION

The Common Core State Standards for Mathematics present unique pitfalls and possibilities for the future of mathematics education. When considering the historical development of the document, the CCSSM embraces aspects of the NCTM standards and introduces new emphases through its inclusion or exclusion of particular concepts. The impact of the standards will ultimately be dependent upon the quality of implementation that exists throughout the United States; therefore, this project created a model for implementation using lesson study. While this model is qualitative in nature, the rationale and findings of the study reveal that lesson study supports careful implementation of the CCSSM when standard pairings are targeted. The use of lesson study supports the development of reflective thinking and knowledge about the CCSSM among teachers. Curriculum specialist Matthew R. Larson (2012) noted that approaches to implementation that develop the knowledge of teachers will be necessary for the CCSSM to positively impact student achievement. From this observation, lesson study represents an easy-to-use model for individuals, schools, or districts seeking to implement CCSSM with student achievement in mind. Along with these considerations, the lesson study provides further support for patience in implementing the CCSSM. Full implementation necessarily includes the practice standards; however, teachers must remember that student development of the skills included in these standards will require dedication, quality opportunities to reason, and time. Finally, the background and findings of this study reinforce the idea of change described in the opening sentences of the introduction. The CCSSM are the product of many changes throughout the history of the American education system. Similarly, the implementation of the CCSSM will require change for some teachers as the skills of the practice standards are integrated into classroom instruction. Along with this aspect of change, the data recovered from lesson studies involving the CCSSM and quantitative studies that develop over the next decade will generate evidence for further changes to the document. Authors W. Gary Martin and Dawn Berk (2001) described this aspect of change best by stating, “the relationship of research with standards inevitably cycles between research helping to shape the next iteration of standards and standards having an impact of future research” (p. 328). The Common Core State Standards are already shaping research in mathematics education even as implementation occurs. This project is evidence of that influence as well as the numerous formal and informal projects occurring throughout the United States. With the data generated from research, the CCSSM may change as better practices are identified and supporters or critics of the standards demand greater coherence; however, the commitment to student excellence will remain. The Common Core State Standards for Mathematics cannot change education, but teachers have the opportunity to creatively implement them, and in doing so will grow in their abilities as instructors. By supporting the standards, administrators can foster collaboration among educators. Most importantly, the standards challenge teachers to develop the reasoning skills of students that will translate to achievement inside and outside of schools.

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APPENDIX A

Lesson 1: Introduction to Constructions

Objectives Aligned to Illinois Common Core Standards

Common Core State Standards for Mathematics:

- G-CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector; and constructing a line parallel to a given line through a point not on the line.*
- Standard for Mathematical Practice 3. Construct viable arguments and critique the reasoning of others.

TLWBAT (the learner will be able to):

- Copy a segment and draw a circle with that radius using a compass and straightedge.
- Bisect a segment and locate its midpoint.
- Construct a perpendicular bisector to a segment.
- Develop an argument for the validity of the perpendicular bisector construction using congruence of triangles.
- Apply knowledge of constructions to construct an equilateral triangle.

Vocabulary

- Construction- A geometric object (special point, line, circle, angle, etc.) that is constructed using tools that establish congruence.
- Compass- A device for completing a construction. A compass has two functions. First, a compass can measure lengths by setting the compass to the length of the desired segment. Second, a compass can make circles and arcs of different radii.
- Straightedge- A tool for drawing straight lines. Remember that one cannot use a straightedge to measure distance (even if the straightedge has measurement markings, like a ruler).
- Bisect- Dividing a finite length into two congruent pieces.
- Perpendicular bisector- A perpendicular to another segment that also bisects the segment.

Instruction Materials & Technology

Whiteboard and markers; compass and straightedge; paper and 6.1 Constructions Handouts; construction animations from Math Open Reference website.

<u>Divergent Questions</u>- higher level thinking questions to be asked throughout lesson

- What does construction mean?
- What shape does a compass make?
- How can you use the basic functions of a compass to copy a segment?
- What's the difference between a bisector and a perpendicular bisector?
- How do you know the construction for a perpendicular bisector is valid? How can you prove it works?
- What are the properties of an equilateral triangle?

<u>Introduction to Lesson</u>

What does construction mean? What do you think of when someone is constructing something?

Typically, we use the word construction when something is being built. Houses, roads, bridges, and other things are what come to mind when we talk about construction. In geometry, a construction refers to something we can make using tools. Sometimes, we will use a compass and straightedge. Other times, we will use other methods. Similar to building a house using wood and power tools, in geometry we will build different objects with tools. We will make lines, angles, shapes, and other interesting figures from geometry. Today we will start with the basics: segments, bisectors, and right angles.

Take out your compass and straightedge. With a piece of paper, move around the compass and move its arm around. What shape does a compass make? How does moving the arm change the shape?

<u>Guided Practice</u>

Note: This lesson relies upon a guided discovery strategy, the use of manipulatives, and scaffolding to teach all students.

A compass has two functions. We've seen that compasses can make circles or parts of a circle (called arcs). Use your compass to make circle B on the worksheet. *Give time to construct circle B.* When we move the arm of the compass, the radius and diameter of a circle change. This effect lets us know that we can use the compass to mark off a length. All we have to do is set the compass to the length of a segment.

The straightedge can only be used to draw straight lines. Even if we are using a ruler as a straightedge, we cannot measure a length with the ruler. Why do you think we do not want to use a ruler to measure? Have you ever noticed that people can get different measurements for something using rulers? Using a compass to mark distances, instead of a ruler, helps avoid these problems.

Let's use what we have discussed to copy a segment. How can you use the basic functions of a compass to copy a segment? Remember what we can do with the compass and straightedge. *Give time to construct* AB.

Now that we have learned the basics, let's start making some more interesting constructions. We can bisect a segment using a compass and straightedge. Does anyone remember what bisect means? To bisect a segment, we will need to make a line that divides the segment in half. This line is called a bisector. What do you think the difference is between a bisector and a perpendicular bisector? *Proceed to guide students through constructing a perpendicular bisector using the handout.*

How do you know the construction for a perpendicular bisector is true? How can you prove it works? Let's work through questions 5-9. *Give time to complete questions 5-9. Scaffold students with questions as needed. After most students have finished, repeat the question above. Make sure students use congruence in their arguments.*

We made an isosceles triangle when constructing a perpendicular bisector. We can also make an equilateral triangle. What are the properties of an equilateral triangle? Keep those questions in mind as you try to construct an equilateral triangle.

Independent Practice

Students will have the opportunity to complete constructions throughout the lesson, but independent practice will occur as students construct an equilateral triangle. Independent practice will also occur through a homework assignment that complements the lesson.

Closure

Constructions are a way to picture different objects from geometry. Based on what we have done today, some constructions are simple and others can be challenging. A good way to think of constructions is to approach them like puzzles. I like to use these steps when I'm doing constructions:

1. What do I want to make? What do I know about what I want to make (angles, sides, etc.)?
2. What am I given?
3. Which construction can I use?
4. When I get the solution, does it make sense (use congruent triangles and what I know about geometry)?

Assessment- the student will be assessed by…

Informal assessment will occur throughout the lesson by using student responses to questions and progress on the handout to gauge comprehension. The teacher will circulate the room when students complete constructions, using questions and a brief survey of student work to measure understanding of concepts and processes. Formal assessment will occur at the end of the unit with a comprehensive exam of constructions.

Modifications/accommodations to meet the developmental & individual needs of diverse learners

In terms of content, the use of various questions throughout the lesson will help students grasp the basic functions of a compass and straightedge. The teacher will use additional scaffolding for specific students as needed when work time is given during the lesson. Attention to word definitions and use of individual questions will accommodate ELL students in the classroom. The tactile nature of geometric constructions will assist student understanding of congruence. The teacher may modify parts of the lesson by having some students measure segments with a ruler to help them see the congruence made with a compass. Students with learning delays will also benefit from the use of construction animations that will serve as an additional guide to the process. Finally, assistive listening devices will be used as needed for a hearing impaired student in the class.

Content-Specific Criteria

A review of parts of a circle may be required. Attention to the concepts and the process will be emphasized. Knowing how and why constructions work to make geometric figures is critical for comprehending the concepts and fulfilling the corresponding Standard for Mathematical Practice that has been paired with this content.

Lesson 1 Reflection

1. How did the lesson go overall?

After teaching two different periods, I think the lesson went well. When teaching the lesson the first time, I covered all the objectives within the hour and students completed the constructions. Participation in answering questions was low when teaching the first time, although I used too many questions with that class. In addition, I worked with groups of students to explain the validity of the perpendicular bisector construction. This choice seemed to leave some students confused, since I was not able to work with each group in the timeframe. Seeing this problem, I chose to run through the proof of the construction with the second class as a whole. The second group of students seemed to understand concepts quicker and I think it was because I explained basic ideas more, while eliminating some questions that were unhelpful during the other class. The second class also completed all constructions stated in the objectives and explained reasoning behind constructions in varying detail. I think the second time teaching the lesson went far better than the first time. Overall, I think the lesson went well.

2. What worked well? What didn't work well?

After teaching both classes, I think including a problem after each construction was introduced was helpful. These problems provided the cooperating teacher and myself time to scaffold individual students, while giving me the opportunity to ask students to explain how they completed constructions. Some students understood the concepts quickly, but other students needed additional assistance. A few students even surprised me by asking if the quadrilateral made during one problem was a square. I did not anticipate students recognizing the shape so easily! Guiding students to the reasoning behind the perpendicular bisector was helpful, although I had better success with this activity during my second class. Students during the first class seemed confused at times and did not grasp the proof as much as I wished, which is probably connected to my pacing and approach to the lesson. I think my pace was too brisk early in the lesson and too slow later in that hour. The varying levels of comprehension in the first class made the lesson seem chaotic, since some students easily understood the ideas and others needed additional scaffolding. Another aspect of the lesson that did not go well was my failure during the first class to explain why students could not use a ruler to find the midpoint. Since I did not explain this point to the first group (which I did with the later class), some students just used a ruler to bisect the segment instead of using the appropriate construction. A final aspect of the lesson that was successful was student explanations of the equilateral triangle constructions. I asked multiple students to explain how they made the triangle using a compass and I was ecstatic to find that most understood the construction was basically making two overlapping circles.

3. How well do you feel the desired Standard for Mathematical Practice (3. Construct viable arguments and critique the reasoning of others) and paired Content Standard (G-CO.12 Make formal geometric constructions) were implemented?

I had mixed success in implementing the practice standard in the lesson, although all students were able to create the constructions related to the content standard. In the first class, I attribute the mixed success of students forming an explanation to my approach to the lesson. I skipped a part of the introduction and chose to guide students through the explanation in groups, which made the lesson awkwardly paced and caused some students to end up confused. During the second class, students seemed to better understand the reasoning behind the perpendicular bisector construction. I think more students developed an explanation for this construction because I helped guide the entire class and used fewer vague questions. Finally, I think the practice standard was implemented effectively in regards to the equilateral triangle construction. While this construction was not directly stated in the paired content standard, multiple students were able to explain their reasoning behind the construction during both classes. One part of the practice standard that was not well evidenced in the lesson was students critiquing other student reasons. This drawback is probably due to my failure to ask students to check their work with other students, along with the students' lack of experience making justifications.

4. Did students seem engaged in activities related to the chosen standards and objectives?

Students seemed engaged in varying levels throughout the lesson. Some students quickly understood the ideas and made constructions, which caused the cooperating teacher and I to provide some students with additional exercises. For example, students in the second class constructed the equilateral triangle quickly, so I had some students create the perpendicular bisectors of the sides to form the circumcenter. Other students seemed to become confused during the lesson, so I had to spend time providing scaffolding. This confusion caused some students to become disengaged and revert to using a ruler to find a midpoint. Students also seemed to exhibit varying levels of engagement when explaining their reasoning for constructions. Some students had difficulty saying how they knew a construction was true, while other students were able to form an argument adequately. Finally, I felt differences in engagement existed between the two classes I taught. This difference is largely related to how I approached teaching the lesson in both classes and how I tweaked the lesson for the second class. In all, the students displayed varying levels of engagements in the activities.

5. What could be done differently to improve the effectiveness of this lesson?

There are many changes I would make to the lesson before teaching it in the future. A major change I made between teaching the lesson to the first and second classes was my approach to questioning. I realized I was using too many questions during the first class, so I rephrased and eliminated some questions when teaching the second time. Using fewer questions that were targeted on key aspects worked well with the second group, so I will make use of fewer (and better) questions in the future. Another change that would improve the lesson for future use would be to discuss the problem of using a ruler to bisect a segment with groups of students. I asked students about this problem in the second class, but I forgot to include this discussion in the first class. I included this question in my lesson plan, although I did not consider it too important. Upon reflection, I should have been more explicit with this aspect of constructions. Finally, I think my presentation of the constructions were too fast at times and too slow at other times. In the future, I will be more aware of pacing in the lesson and have a better idea about how time needs to be allocated. In all, there are multiple aspects of the lesson I would change before teaching it in the future.

Lesson 2: More Geometric Constructions

Objectives Aligned to Illinois Common Core Standards

Common Core State Standards for Mathematics:

- G-CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector; and constructing a line parallel to a given line through a point not on the line.*
- Standard for Mathematical Practice 1. Make sense of problems and persevere in solving them.

TLWBAT (the learner will be able to):

- Copy an angle using a compass and straightedge.
- Analyze the problem of constructing a parallel line through a given point (i.e. forming a transversal, recognizing corresponding angles, etc.).
- Create a line parallel to a given line through an external point.
- Construct a perfect square using parallel and perpendicular.
- Construct a rectangle and parallelogram.

Vocabulary

- Angle- A shape formed by two intersecting lines.
- Parallel- Two (or more) lines that never intersect.
- Transversal- A line that intersects parallel lines.
- Corresponding angles- Angles that are equal in measure and located in corresponding positions around the transversal of parallel lines.
- Square- A quadrilateral where all sides and angles are congruent.
- Rectangle- A quadrilateral where all angles and opposite sides congruent.

Instruction Materials & Technology

Whiteboard and markers; compass and straightedge; paper and 6.2 Constructions Handouts; construction animations from Math Open Reference website.

Divergent Questions

- How can you make angles with a straightedge?
- What's the best way to define the word angle?
- Are the triangles made when copying an angle congruent? How are they congruent?
- If you have two parallel lines with a transversal and know the measure of one of the angles, how can you find the other angle measures?
- Which pairs of angles are congruent in parallel lines?

Introduction to Lesson

Yesterday, we explored the basic constructions. What construction did we prove yesterday? *Wait for responses.* The perpendicular bisector construction is helpful for dividing segments, but we can use it for other reasons. If I wanted to make a right angle, all I would do is make a perpendicular line using our construction. Today, we'll be exploring constructions involving angles and connecting them to parallel lines.

Guided Practice

First, take a straightedge and pencil. Try to make an angle with these tools. *Give students about a minute to make an angle.* How can you make angles with a straightedge? *Field answers and lead students to notice an angle forms from two intersecting lines.* What's the best way to define the word angle?

If we have an angle and want to copy it, our first step is just to draw another line. Since we said an angle is just two intersecting lines, we naturally have to start with a line to make an angle.

Guide students through rest of construction for copying an angle using 6.2 Constructions handout. Check for understanding. Point out congruent segments being made when copying angle. Notice we have three points in our angles when we complete the construction. Let's join the two unconnected points in each angle with a line to make triangles. These segments are also congruent (we can check with our compass if we want to be sure). Are the triangles made when copying an angle congruent? How are they congruent? *Answers will be brief, but will help measure comprehension.* Based on what we've just discussed, copying an angle can be thought of making two congruent triangles.

Keep in mind copying angles as we move to our next construction. If we are given a line and a point off the line, we can construct a parallel through the point. Before we work through the construction, let's figure out how to do it. If you have two parallel lines with a transversal and know the measure of one of the angles, how can you find the other angle measures? *Discuss corresponding angles with students.* With this idea, if we somehow had an angle for one line we could make a parallel line by copying the angle.

If you want to make an angle for the line in the problem, what will you have to do? Think back to the start of the lesson. *Give time to answer.* By drawing another line through the given line, you can make an angle. If we want a parallel line through the point off the line in the figure, then drawing a line that includes that point seems natural enough. Looking at what we have now, we have a line with a transversal through a point. If we want a parallel line, all we have to do is copy an angle we made to the part of the transversal using the point (remind students of parallel line angles and ask students a question rather than tell them to copy an angle). We've simplified the problem. Which pairs of angles are congruent in parallel lines? Let's try to solve it differently by copying different angles. *Have different tables copy different angles to make parallel lines (ex. Alternate Interior Angles, Corresponding Angles, Alternate Exterior Angles). Check for understanding and have students compare results of using different pairs.*

<u>Independent Practice</u>

After completing the practice above, close the topics and remind students of making perpendicular lines. Students will work on constructing a square and parallelogram independently, receiving hints and assistance as needed.

<u>Closure</u>

Looking at what we've explored yesterday and today, constructions are connected to one another. Often a construction we did previously is used to complete another construction. With the basic ideas and constructions, we'll be able to make more complex constructions. A key thing to remember when making a construction is to consider the properties of what we want to make, like we did with parallel lines. A lot of times these properties help us figure out how to make a construction.

<u>Assessment</u>

Informal assessment will occur throughout the lesson using student responses and progress on constructions. The completion of constructions included in objectives will be another informal assessment. Students will be assessed formally using homework and a cumulative test at the end of the unit.

Modifications

Additional scaffolding will be required for some students in the classroom. The inclusion of a step-by-step guide to the constructions will assist students with comprehension problems. The square and parallelogram constructions may be modified for some students by providing a line and perpendicular rather than nothing. Questions will be asked of students and directions will be clarified for students who are ELL or have a hearing loss. Written directions will also be posted to assist these students. A computer animation of a construction will play during independent work time to remind students of previous constructions. Finally, assistive technology will be used as needed.

Lesson 2 Reflection

1. How did the lesson go overall?

After discussing with the cooperating teacher and reflecting upon the two classes, I think the lesson went well overall. Similar to the first lesson, the students displayed varying levels of understanding when copying angles and creating parallel lines. Some students caught on quickly and even began constructing the chosen quadrilaterals; however, most students were only able to complete the parallel line construction. This pattern occurred in both classes. In hindsight, the quadrilaterals could have served as a separate lesson the following day and freed some time to explain the parallel line construction in more detail. The cooperating teacher and I provided scaffolding for students as needed; thus, every student was able to copy an angle and make a parallel line successfully. While student reasoning for constructions varied in detail, most students understood why copying an angle is just making congruent triangles. Overall, I think the lesson was an improvement over the first lesson and was a good experience.

2. What worked well? What didn't work well?

During the lesson, a number of factors worked well and other factors needed changes. An approach that worked well during both classes was using congruent triangles to prove the angle copying construction. In particular, students in the second class picked up on why the compass was being used to mark distances (to make equal sides of a triangle). An aspect of the lesson that needed adjustment was the number of examples that I used to guide students. In the first class, I realized that students needed multiple examples to become familiar with the construction. With this realization, I included three additional examples during the second class and noticed an improvement in understanding. Another aspect of the lesson that worked great was my scaffolding for the parallel line construction. I noticed that most students had trouble seeing why the angle needed to have a vertex on a particular point. To explain this aspect of the problem, I had students picture laying the given angle on top of the point. With that frame of mind, most of the students I spoke to were able to see the solution. A final aspect of the lesson that did not work well was the design of the worksheet. While including the construction steps was helpful for students, placing blank boxes next to the steps confused students. By including the boxes, some students tried duplicate each step in the blank boxes instead of working with one sketch. In all, different factors worked well in the lesson and others required adjustment and rethinking for future use.

3. How well do you feel the desired Standard for Mathematical Practice (1. Make sense of problems and persevere in solving them) and paired Content Standard (G-CO.12 Make formal geometric constructions) were implemented?

The content standard was effectively implemented during the lesson. While students did not complete the quadrilateral constructions, these complex problems went beyond the suggested constructions in the related standard. Students managed to copy an angle and construct a parallel line, which fulfilled most of the lesson objectives that reference the content standard. In terms of the chosen practice standard, I think students displayed the aspects of the standard at varying levels. Some students used the idea of copying angles to make a parallel line, but many students were confused about how to approach the problem. With appropriate scaffolding, students completed the constructions; however, the perseverance part of the practice standard seemed nonexistent among students who gave up before trying the problem independently. Overall, the cooperating teacher reminded me that students in the class have less experience reasoning with problems. This limited exposure explains the reactions of some students who did not understand why the class was learning constructions. In all, the content standard was implemented effectively and the practice standard was embodied in the thought process of some of the students.

4. Did students seem engaged in activities related to the chosen standards and objectives?

Students were engaged throughout the lesson by creating constructions outlined in the objectives of the lesson. A challenge that existed during the lesson was supporting student attempts to reason with the problem. For example, some students looked at the parallel line construction and did not attempt the problem without support from the cooperating teacher or myself. While subsequent attempts to have students see the problem as copying an angle were successful, I felt that some students did not persevere in making the construction independently. In all, students were engaged in objectives related to content and displayed mixed success engaging in sense making.

5. What could be done differently to improve the effectiveness of this lesson?

A number of changes to the lesson will improve its effectiveness. The simplest change that would improve the lesson would be to make the quadrilateral constructions a separate lesson, providing more time to work on the other constructions listed in the objectives. In addition, I would include more examples for students to gain experience with the construction. I might also consider making a "challenge sheet" for students that understand and complete constructions quickly. This activity may include other constructions from future lessons or be some sort of puzzle with constructions (such as constructing the altitudes of a triangle and finding the orthocenter). Another change to the lesson that would be helpful would be to rearrange to order of the parallel line construction. Instead of beginning with connecting the construction to prior problems, it would be better to place the explanation later in the lesson after working through a few examples with the students. Finally, a change I will make to the lesson will be to take a moment to discuss why constructions are worth learning. While I helped students see the connection of constructions to congruent triangles, I did not mention why constructions are useful for understanding congruence. This small aspect would have helped students who did not see the point of the topic. With proper adjustments, the lesson will be improved for future use.

Lesson 3: Paper Folding- A Different Way to Make Constructions

<u>Objectives Aligned to Illinois Common Core Standards</u>

Common Core State Standards for Mathematics:

- G-CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector; and constructing a line parallel to a given line through a point not on the line.*
- Standard for Mathematical Practice 5. Use appropriate tools strategically.

TLWBAT (the learner will be able to):

- Bisect a segment and locate its midpoint using tracing paper.
- Construct a perpendicular line and perpendicular bisector to a segment using tracing paper.
- Bisect an angle using tracing paper/compass and straightedge.
- Create a line through a point that is parallel to another line using tracing paper.
- Construct a square with tracing paper.
- Use tools (straightedge, compass) to verify the constructions made by paper folding.

<u>Vocabulary</u>

- Construction- a geometric object (special point, line, circle, angle, etc.) that is constructed using tools that establish congruence.
- Representation- a way of present information or ideas to communicate understanding.

- Verify- checking the correctness of a process.
- Tracing Paper- thin paper that allows one to see an image on both sides of the paper.

Instruction Materials & Technology

Whiteboard and markers; compass and straightedge; tracing paper; 6.3 Constructions handout; ELMO magnification system.

Divergent Questions- higher level thinking questions to be asked throughout lesson

- How can we make lines on tracing paper without using a pencil and straightedge?
- Why does folding the paper bisect the segment? How can we check?
- Are the constructions made with paper always identical to ones made with a compass and straightedge? What are some reasons they might be slightly different?
- What is a representation?

Introduction to Lesson

Over the past two lessons, we've figured out how to make constructions and why they work. What are some constructions we've made? *Wait for student answers*. Up to this point, all the constructions have been made with a compass and straightedge. What if I didn't have a compass? If I just had paper, would I be able to still make a construction?

Guided Practice

Note: Guided practice will occur throughout the lesson as students complete the constructions using tracing paper. Strategies used throughout the lesson include guided discovery, collaborative learning, scaffolding, and manipulatives (tracing paper).

Take a piece of the tracing paper and place it on top of your handout. As you can see, the paper is thin enough to see the handout. Copy CD onto the paper with a pencil. Looking at the tracing paper, how would we be able to make a line that bisects CD? Think about folding a piece of paper in half. *Give students a moment to think.* When we fold paper in half, we line up the edges. If we look at our tracing paper, we can fold the segment in half by lining up the points.

Provide a moment to fold paper. When you look at paper, you see that the crease in the paper bisects the segment. We can think of the crease as a line. How can we make lines with the tracing paper without a pencil? *Wait for answers.* We can see that the segment looks bisected. Why does folding the paper bisect the segment? How can we check? *Discuss.* Grab a compass and straightedge. Using what we learned the other day, bisect the segment with the compass and straightedge. *Pause for a moment for students to complete task. Circulate and scaffold as needed.*

Looking at the fold and the marks your compass made, is the bisector in the same place? Are the constructions made with paper always identical to ones made with a compass and straightedge? What are some reasons they might be slightly different? *Discuss.* Basically, the paper folding is a different representation of the construction. What is a representation? *Field student answers.* In math, a big idea is being able to see different perspectives on a problem. As we go through the following constructions, think about the constructions the other day and how we could represent them with folding.

Now, let's move to making a perpendicular bisector. What do you remember about bisecting a segment from the other day? Was the bisector made with the compass perpendicular? *Have students provide brief responses.* Since bisecting the segment made the perpendicular bisector, we've actually already made one for segment CD. Let's make a perpendicular bisector for EF. After you complete your fold, draw a line following the crease and check your work with a compass. *Allow students to complete construction. Circulate and field questions/provide help as needed.*

For the next two constructions, work with your tables to bisect the angle and make a parallel line using only paper folding. An angle bisector divides an angle in half. We did not construct this line with a compass, but I'll help you with it after you do it with paper folding. For the parallel line construction, it might be best to think about making a perpendicular line through the point. After making your constructions, check that your work makes sense using a compass and seeing if the fold looks right. *Provide time for students to work on constructions independently. Guide students as needed. Remind students that they folded a segment to bisect it, so they can do something similar for angles.*

Independent Practice

Students will complete the angle bisector and parallel line constructions in groups, but assistance from the teacher will be minimized to encourage independence. In addition, each student will make his/her own constructions, so every student will have a unique product. Finally, students will complete the square construction independently.

Closure

Looking at what we did today, the only new construction we learned was bisecting an angle. We already bisected segments, made parallel lines, and constructed a square. We made these constructions with paper to see another way to think about constructions. Folding paper to make a bisector is the same as using a compass and straightedge. The big idea is that by knowing one way to do a problem helps us see another way to do the problem. Knowing different ways helps us become experts with constructions! Using what we already know about constructions, we can start looking at constructions in different ways and prove that other ways work.

Assessment- the student will be assessed by...

Informal assessment will occur throughout the lesson by using student responses to questions and progress on the handout to gauge comprehension. The teacher will circulate the room when students complete constructions, using questions and a brief survey of student work to measure understanding of concepts and processes. Formal assessment will occur at the end of the unit as a comprehensive exam of constructions.

Modifications/accommodations to meet the developmental & individual needs of diverse learners

The ELMO magnification system will be used to have students follow along with bisecting the segment. This visualization will assist students who need additional support with the construction. The nature of paper folding makes the entire lesson hands-on, accommodating students with learning disabilities. During independent practice, additional instruction and explanation will be provided to students as needed. In particular, ELL and hearing impaired students may need clarification on instructions. Finally, students will be allowed to refer to previous handouts for creating constructions with a compass. Allowing these resources to be used will support student comprehension of the topic and develop familiarity with constructions that will be assessed later in the unit.

Teacher Reflection:

To be completed after lesson is taught.

Content-Specific Criteria

This lesson will serve as an informal measure of student understanding of basic constructions. By encouraging the use of a variety of tools to check paper folding, students develop additional experience generating constructions. Teaching students to bisect an angle with a compass may occur with the whole class or in groups, depending on what seems to work best. The square construction may need a little clarification for students, so remind them of the properties of equal sides and right angles.

Lesson 3 Reflection

1. How did the lesson go overall?

I think the lesson went well. Most students were able to complete the constructions independently or with scaffolding from the cooperating teacher and myself. After noticing the confusion in the first class that resulted from including compasses, I chose not to use the device during the second class. Both classes displayed varying levels of understanding and some students did not complete the square construction during the class period. In spite of these drawbacks, many students seemed to catch onto the paper folding faster than making constructions with a compass and straightedge. Overall, I think the lesson went well.

2. What worked well? What didn't work well?

Using the paper to make constructions seemed to work well. On the whole, students formed a better understanding of the parallel line construction by using the paper folding. I also think using the ELMO magnification camera was a good way to guide students. In addition, I think leaving time for students to work on constructions independently was helpful. The time provided me the chance to scaffold students as needed and ask students how they made a construction. Some aspects of the lesson that did not work well were the ordering of the problems and the pace of my instruction. After discussing with the cooperating teacher and university supervisor, we concluded that the square construction may have been a better entry point into the lesson than saving it for last. Students could have started with a completed square and proved it was a square using folds (making a natural launching point for other constructions). Along with the order of the lesson, I felt that my pace was too fast at times. Because of my pace, I had to use more scaffolding with individual students.

3. How well do you feel the desired Standard for Mathematical Practice (5. Use appropriate tools strategically) and paired Content Standard (G-CO.12 Make formal geometric constructions) were implemented?

I think the desired content standard was implemented, but the chosen practice standard was not implemented effectively. While students completed constructions, my choice to discard the compass part of the lesson basically eliminated student use of multiple tools. This choice was mainly due to the confusion seen in the first class; however, the second class might have been able to use the tools effectively after witnessing student success with making more difficult constructions. While the students were not able to use various forms of technology, I discovered that many students seemed more comfortable explaining their reasoning (relating to Standard for Mathematical Practice 3. Make viable arguments and critique the reasoning of others). A key point in the lesson that illustrated this standard was when a student found an alternative way for making a parallel line. While the student's method was similar to mine, he used fewer folds to made a valid construction. I had the student demonstrate his process for the class and most students agreed it was an easier method. Overall, I feel the chosen content standard was implemented effectively and the paired practice standard was a poor fit for the lesson.

4. Did students seem engaged in activities related to the chosen standards and objectives?

Students displayed multiple levels of engagement with the lesson. For the most part, students seemed more engaged compared to making constructions with a compass. Students had difficulty learning how to use the compass effectively, whereas the paper folding was completely tactile in nature. Despite this success, some students got lost during the lesson. Some students made an incorrect fold that threw off the construction, which caused confusion. Other students did not see a step to a construction that was completed by the teacher, causing them to skip steps. Finally, I think this lesson had different levels of engagement because some students completed the problems quickly and others needed support. I think if I had some extension problems for some students, they may have been more engaged in the lesson.

5. What could be done differently to improve the effectiveness of this lesson?

There are a number of changes I can make to the lesson to improve its effectiveness. In the future, I would probably rearrange the problems as discussed earlier to make the lesson more cohesive and reasoning-based. Another change that will help improve the lesson will be to have students become more familiar with the compasses earlier in the week, so that the tools could be incorporated into the lesson without confusion. Along with this change, I would leave the angle bisector construction more open-ended to see if students could create the construction independently. Finally, a change to the lesson that would engage all students effectively would be to use smaller groupings of students to teach the lesson. While this model is not possible in every classroom, I think the approach would have been helpful in the class that included a special education teacher, the cooperating teacher, and me. With three teachers in the room, groups could have been a better approach to the lesson and I will consider that model in the future. In all, various changes to the lesson could improve its effectiveness in future use.