

A graph G is a mathematical object that consists of a set of vertices $V(G)$ and set of edges $E(G)$ such that an edge $e \in E(G)$ connects two distinct vertices in $V(G)$. An Italian dominating function $f : V(G) \rightarrow \{0, 1, 2\}$ assigns weights of 0, 1, or 2 to each vertex such that each vertex $v \in V(G)$ with a weight of 0 ($f(v) = 0$) has at least two neighbors $u_1, u_2 \in V(G)$ with weights of 1 ($f(u_1) = f(u_2) = 1$) or at least one neighbor $w \in V(G)$ with a weight of 2 ($f(w) = 2$). In short, if v is assigned a weight of 0, then $\sum_{w \in N(v)} f(w) \geq 2$ for $N(v)$ denoting all vertices adjacent to v . An Italian dominating family is a distinct set of Italian dominating functions $\{f_1, f_2, \dots, f_n\}$ such that the sum of the weights of some vertex $v \in V(G)$ across all Italian dominating functions in this set does not exceed 2 for every vertex in the graph of G . In other words, an Italian dominating family must satisfy $\sum_{i=1}^n f_i(v) \leq 2$ for all $v \in V(G)$. The maximum cardinality of the Italian dominating family for G is called the Italian domatic number of G , denoted $d_I(G)$. In this report, we determine the Italian domatic number across varying families of Cartesian products, including $T_1 \square T_2$ for two trees that are not both stars, $C_{3r} \square C_{3s}$ for $r, s \geq 1$, $C_4 \square C_{3r}$ for $r \geq 1$, and $C_{5r} \square C_{5s}$ for $r, s \geq 1$. We also classify the Italian domatic number for all trees based upon the presence of a specific configuration in the tree.